Math 1112
Unit 1: Numbers and Their Relations

Introduction to Module 1

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Sections below are procedural and won't be tested, but may be useful for you.

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Intro to HNP web-application and expectations for this unit

This booklet contains a snapshot of the material that is required for mastering numbers and their relations sufficiently for success in the rest of the course and subsequent courses. The material is not new to you, but you may be rusty in some of the concepts.

Numbers are both simple and complicated in that they exist as objects that we can play with and study (how much larger is 3.1 than 1.7), but their main job is to represent quantities of concrete objects (3 cows can produce up to 90 litres of milk per day).

In this course I have separated the two ways numbers exist, and the material in this unit focuses on reviewing your ability to think about the basics of numbers and their relations as numbers (no cows and no milk).

The HNP web-application aims to help individuals build up their skills in thinking about numbers and their relations by providing practice questions scaled up from simple to more complex.

How to earn mastery points in this unit (your teacher may choose not to use mastery based learning):

Complete the online review assignments and tests 1 & 2 available through the HNP web app (bit.ly/henupr). To obtain mastery you need to complete the tests with a score of 80%. The assignments are designed to help you practice the concepts before doing the tests. If you do the tests first and achieve >80% then you don't need to do the assignments, if you don't achieve 80% then I will strongly suggest that you do the assignments in the areas you are struggling with. Instructions will be provided in class and will be available on D2L.

What do I do with the booklet?

This booklet provides extra resources that can be used for a different kind of practice and for a quick snapshot of what is in this module of study.

The booklet also contains some extra material which is not in the HNP web-app for those who want to review some of the procedural math skills, and it has a few readings that aim to help guide you get used to studying math in a college setting.

Introduction to Numbers as objects of study

Though Numbers and other abstract objects that arise from them are the traditional domain of mathematics, we don't often spend time actually thinking about them and their relations.

Most of our focus tends to go to developing the ability to do procedures (operations) with numbers quickly which then help with other procedures (operations) in basic algebra and beyond.

In this module of the course we will slow down our thinking a bit and aim to get you re-acquainted with numbers and their relations. You will notice a few things about the way this module is set up.

- In many questions exercises and lessons calculators will not be of much use to you. That is on purpose as the goal is to get you to be comfortable with numbers, with what they mean as abstract objects and how they relate to each other.
- Given that this is a course for Health Science students you will not be playing around with negative numbers, as they do not play a meaningful role in health care research.
- You won't see the word 'fraction' anywhere in this course (except in this sentence 😊) as a very conscious choice has been made to represent fractions as numbers that are 'between whole numbers'. In formal mathematics numbers between whole numbers (which includes whole numbers themselves) are called rational numbers (can be written as one whole number 'a' divided by another whole number 'b' represented as a/b). You will spend time getting really good at reading, comparing and relating both large and small numbers between whole numbers.
- For the booklets I have only provided you with 'answers' to the questions without any explanations or justifications. In the HNP web-app we have developed an extensive practice data base of questions with explanations.
- I will not be formally teaching this module as the content here is meant to represent a review of skills important before the start of the course.

Signs of comparison with Whole Numbers

Exercise 1: Put True or False beside each of the following

a. 16 is closer to 0 than to 100
b. 45 is closer to 50 than to 30
c. 234 556 is closer to 230 000 than to 240 000
d. 674 is closer to 600 than to 700

Exercise 2: Which of the following is the number 37 closest to? Circle the correct value.

a. 45 b. 35 c. 40 d. 0

Exercise 3: Which of the following is the number 374 closest to? Circle the correct value.

a. 450 b. 350 c. 400 d. 0

Exercise 4: Put True or False beside each of the following

a. $16 > 5 \times 3$
b. $144 = 11^2$
c. $3.14 \times 10^7 < 3.14 \times 10^6$
d. $77/11 > 4+3$

Exercise 4: Place the correct signs of comparison between the following pairs of numbers. greater than (>); less than (<) or equal (=).

a. $3 + 4$ __ $3 + 7$ b. 33 ____ 7×6 c. 8×9 __ $4 \times 2 \times 3 \times 3$

d. 44/4 __ 8 e. $22 + 17$ __ 3×13 f. 6×7 ___ 7×3

g. 10^3 ___ 10^2 h. 7.1439×10^3 ____ 7 000 i. 10^4 ___ 31 405

Arithmetic with Whole Numbers

Below are a series of exercises that will help you refresh your feel for working with whole numbers and remind yourself of the sign of equality and signs of inequality. It is very important that you do all of the questions in the intuition sections **without using a calculator**.

Reminder: $>$ is a symbol for 'greater than' $7 > 3$; $<$ is a symbol for 'less than' $3 < 7$;

Exercises:

1. Fill in the blanks for the following equations with correct values.

a) $3 + 4 =$ ___ $+ 2$ b) $33 +$ ____ $= 7 \times 6$ c) $8 \times 9 = 4 \times 3 \times$ ___ $\times$ ____

d) $44/4 = 8 +$ ___ e) $22 + 17 = 3 \times$ ____ f) $6 \times 7 =$ ____ $\times$ ____ $\times 3$

g) $10^3 =$ ___ $\times\ 10^2$ h) $7.1439 \times\ 10^3 =$ _____ i) _____ $\times 10^4 = 31405$

2. Comparisons: Fill in the blank with one of the following symbols: $<$, $>$ or $=$

a) 3×72 ___ 372 b) 13×20 ___ 200 c) 10×200 ____ 15×100

d) $945+377$ ___ 13×100 e) 75×7 ___ 4000 f) 12×12 ___ 11×15

g) $37/5$ ___ 6 h) $1000/10$___ 100 i) $7653/ 12$ ___ 1000

3. Prime numbers: A prime number is a number that is divisible only by itself and 1 (for example 17 is a prime number as there are no numbers other than 17 and 1 that divide evenly into 17).

a) Find at least 2 prime numbers between 20 and 30

b) Find at least 3 prime numbers between 50 and 70

Numbers Between Whole Numbers

Whole numbers are numbers that come from a recognition that we can count from 0 up to infinity by ones. They are represented in 'set' notation as follows W = {0,1,2,3,4,5….} and other than 0 are ones that are generally considered as 'natural' in some way even though they are abstract object.

Zero is a bit different in that it isn't really natural, because it represents nothing as if it was something. The more we think about 0 as a number the weirder it becomes… luckily we can do a lot of good things mathematically without really getting into that weirdness.

Just as zero became a really important concept in mathematics history, so did numbers between whole numbers, which includes numbers like 0.005; $\frac{1}{17}$; 54.39; and 100. Each of those is a number between whole numbers… more formally each of those numbers can be written as a ratio of two numbers a/b: 0.05 = $\frac{5}{100}$, $\frac{1}{17}$ already is in ratio form, 54.39 = $\frac{5439}{100}$, 100 = $\frac{100}{1}$.

In the HNP web-application you will see a module dedicated to numbers between 0 and 1; and a separate module dedicated to numbers between whole numbers; which includes rates that are >1 (e.g. 35.14 = $\frac{3514}{100}$).

In this booklet you will get a small taste of the work that is ahead of you.

Numbers on a line

Not all numbers are whole numbers. Some are integers (i.e. include negatives) and some represent parts of a whole (called rational numbers – or numbers between whole numbers). Below is a worksheet to help you remind yourself of numbers that represent parts of a whole. Again you will not need a calculator.

Exercises:

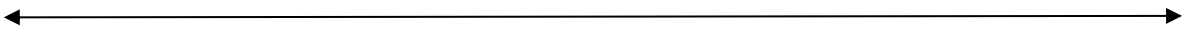
1. Write the following sets of numbers in order from smallest to largest

   a) 5, 5555, 555, 55 b) $\frac{1}{5}$, $\frac{1}{555}$, $\frac{1}{55}$, $\frac{1}{5555}$

   c) $\frac{1}{3}$, 0.17, $\frac{1}{4}$, 0.5 d) 0.001, 10^{-1}, 3^{-1}, $\frac{1}{2}$

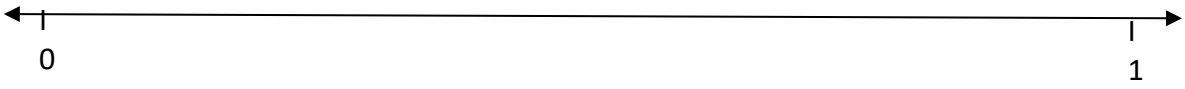
2. Mark the following numbers on the number line below (note that you will need to figure out where to place the zero yourself.

   a) 0 b) 3 c) 17 d) –2 e) –4



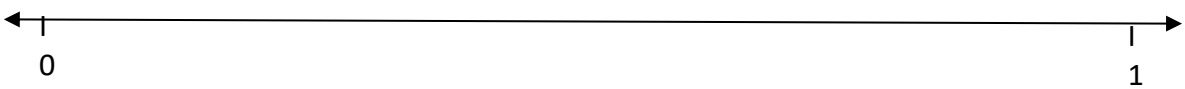
3. Mark the following points on the number line below.

   a) 0.4 b) 0.017 c) $\frac{1}{10}$ d) $\frac{1}{5}$ e) $\frac{5}{8}$ f) 0.05



4. Mark the following points on the number line below.

   a) 0.9 b) 0.01 c) $\frac{1}{10}$ d) $\frac{1}{9}$ e) $\frac{99}{100}$ f) $\frac{8}{9}$



Multiple representations of numbers between 0 and 1

Being able to represent numbers between 0 and 1 in multiple formats automatically is important in mathematics and statistics. This worksheet will help you strengthen your ability to do this, but it is important that you work through it slowly and do not use a calculator.

Thinking tip: when you see a number in a/b format you can think of it in multiple ways. $\frac{2}{7}$ (for example) can be a ratio/rate (2 parts out of 7) a statement of division (2 divided by 7 = 0.857) or a number on a number line (between 0 and 1 and closer to 0)

1. Which of the following is equivalent to 0.01? Circle all correct answers.

   $\frac{1}{100}$ 1% 0.10% 0.10

2. Which of the following is equivalent to 0.017? Circle all correct answers.

   $\frac{17}{100}$ 17% 0.17% 0.17 $\frac{17}{1000}$ $17 \times \frac{1}{100}$

3. Which of the following is equivalent to $\frac{1}{3}$? Circle all correct answers.

   25% 33.333% 1.33% 0.333 0.30

4. Which of the following is equivalent to 0.03? Circle all correct answers

   3/100 3% 0.30% 0.30

5. rewrite the following % as divisions by 100.

   a) 33% = $\frac{\quad}{100}$ b) 2% = $\frac{\quad}{100}$ c) 2.7% = $\frac{\quad}{100}$ d) 127% = $\frac{\quad}{100}$ e) 0.35% = $\frac{\quad}{100}$

6. rewrite the following decimals as divisions by 100.

   a) 0.22 = $\frac{\quad}{100}$ b) 0.25 = $\frac{\quad}{100}$ c) 0.05 = $\frac{\quad}{100}$ d) 0.34 = $\frac{\quad}{100}$ e) 1.28 = $\frac{\quad}{100}$

Multiple representations of numbers between 0 and 1 (continued)

7. rewrite the following as decimals.

a) $\frac{30}{100}$ b) $\frac{100}{100}$ c) $\frac{350}{100}$ d) $\frac{5}{100}$ e) $\frac{1000}{100}$

8. rewrite the following as percents.

a) $\frac{3}{10}$ b) $\frac{1}{4}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$ e) $\frac{1}{20}$ f) $\frac{19}{20}$

9. rewrite the following % as decimals.

a) 33% b) 2% c) 27% d) 127% e) 35% f) 152%

10. rewrite the following decimals as percents.

a) 0.22 b) 0.25 c) 0.05 d) 0.34 e) 0.95 f) 1.28

11. What number added to the following make a result equal to 1. Write your answer as a rate in a/b form. (e.g. for 0.03 the answer would be $\frac{97}{100}$ since 0.03 + 0.97 = 1.00 and 0.97 = $\frac{97}{100}$)

a) $\frac{1}{3}$ b) $\frac{2}{7}$ c) 0.17 d) $\frac{5}{6}$ e) $\frac{1}{1000}$

Compare numbers between whole numbers.

Exercise 1: Put True or False beside each of the following

a. 0.16 is closer to 0.1 than to 0.2

b. 4.99 is closer to 5.000 than to 4.991

c. 0.017 is closer to 0.05 than to 0.01

d. $\frac{1}{5}$ is closer to 25% than to 5%

Exercise 2: Which of the following is the number 3.7 closest to? Circle the correct value.

a. 4.5 b. 3.5 c. 4.0 d. 0

Exercise 3: Which of the following numbers is closest to 1? Circle the correct value.

a. 0.50 b. 64% c. $\frac{1}{5}$ d. 1.3 e. 0.47

Exercise 4: Put True or False beside each of the following

a. $1.6 > \frac{3}{2}$ b. $\frac{7}{8} < \frac{3}{4}$ c. $\frac{1}{5} < 10\%$ d. $\frac{19}{20} = 95\%$

Exercise 5: Circle the larger rate/ratio

a. $\frac{22}{35}$ or $\frac{3}{5}$ b. $\frac{33}{72}$ or $\frac{7}{18}$ c. $\frac{3}{17}$ or $\frac{8}{51}$ d. 7:9 or 3:4

Exercise 6: Place the correct signs of comparison between the following pairs of numbers. greater than (>); less than (<) or equal (=).

a. $\frac{1}{2}$ ___ 45% b. $\frac{1}{7}$ ___ $\frac{1}{2}$ c. $\frac{1}{10}$ __ 0.001

d. $\frac{2}{3}$ — $\frac{3}{4}$ e. 100% ___ $\frac{100}{100}$ f. $\frac{1}{5}$ — $\frac{1\,000}{5\,000}$

Arithmetic with small numbers: conceptual approach

Make sure to do these questions intuitively, without calculator nor procedural techniques

For example: in evaluating 0.34 +0.41one approach you can take is to think about 34 cents + 41 cents – then 3 dimes and 4 pennies + 4 dimes and a penny (even though we don't have pennies ☺) leading to a total of 7 dimes and 5 pennies. i.e. 0.34 + 0.41 = 0.75. This is just one possibility.

1. Evaluate the following using two approaches:
   a. 0.25 + 0.31 b. 0.01 + 0.036 c. 0.045 – 0.022

2. Evaluate the following using two approaches.
   a. 0.1×32 b. 0.01×32 c. 0.02×32 d. 0.25×32 e. 0.9×10

3. Evaluate the following and draw visuals below to represent the addition or subtraction that you have completed:
   a. $\frac{1}{4} + \frac{1}{2}$ b. $\frac{1}{5} + \frac{3}{5}$ c. $\frac{3}{5} + \frac{3}{5}$ d. $\frac{1}{2} + \frac{1}{4} + \frac{3}{8}$

4. Evaluate the following. (practice thinking of multiplication with small numbers like $\frac{1}{2} \times 5$ as $\frac{1}{2} of\ 5$) Draw a sketch for each.
   a. $\frac{1}{3} \times 6$ b. $\frac{1}{2} \times 3$ c. $\frac{1}{2} \times \frac{2}{3}$ d. $\frac{2}{3} \times \frac{1}{2}$

Arithmetic with small numbers: continued

5. Evaluate the following. (practice thinking of division with small numbers like $\frac{1}{2}/5$ as $Take\frac{1}{2}and\ divide\ into\ 5\ pieces.\ How\ 'big'would\ each\ piece\ be?\ or\ How\ many\ 5's\ fit\ into\ \frac{1}{2}$) Draw a sketch for each.

   a. $\frac{\frac{1}{2}}{3}$ b. $\frac{\frac{2}{3}}{2}$ c. $\frac{\frac{3}{4}}{2}$ d. $\frac{\frac{1}{10}}{5}$

6. Evaluate the following. (practice thinking of division with small numbers like $5/\frac{1}{2}$ as $How\ many\ \frac{1}{2}s\ fit\ into\ 5?$) Draw a sketch for each.

   a. $\frac{4}{\frac{1}{2}}$ b. $\frac{5}{\frac{1}{3}}$ c. $\frac{6}{\frac{1}{2}}$ d. $\frac{6}{\frac{2}{3}}$

7. Evaluate the following. (practice thinking of division with small numbers like $\frac{\frac{1}{2}}{\frac{1}{3}}$ as $How\ many\ \frac{1}{3}s\ fit\ into\ \frac{1}{2}$? You will need to draw a sketch for each to figure these out.

   a. $\frac{\frac{1}{2}}{\frac{1}{4}}$ b. $\frac{\frac{2}{3}}{\frac{1}{2}}$ c. $\frac{\frac{3}{4}}{\frac{3}{4}}$ d. $\frac{\frac{1}{10}}{\frac{1}{5}}$ e. $\frac{\frac{1}{5}}{\frac{1}{10}}$

Multiply and divide by orders of 10

Multiplying and dividing with orders of 10 automatically is important in mathematics and statistics. This worksheet will help you strengthen your ability to do this, but it is important that you work through it slowly and do not use a calculator. Some of the challenges I present to you will require that you are quite comfortable with exponent notation and especially negative powers. Exercise 1 should help you review that.

Exercise 1: rewrite the following whole numbers and decimals as powers of 10. (e.g. $100 = 10^2$; $0.1 = 10^{-1}$)

a) 100 b) 1 000 000 c) 0.01 d) 0.0001 e) 10 f) 0.00001

Exercise 2: evaluate the following– express your answers in decimal format. Do not round off.

a) 10×3 b) 10×3.1 c) 10×0.1 d) 0.31×10 e) 10×0.001 f) 10×314

g) 100×3.14 h) 10×3.145 i) 100×0.314 j) 0.001×100 k) 100×31.4 l) 100×1.01

Exercise 3: evaluate the following – express your answers in decimal format. Do not round off.

a) 3/10 b) 3.1/10 c) $3 \times \frac{1}{10}$ d) $3.1 \times \frac{1}{10}$ e) 314 /10 f) 0.1 /10

g) $\frac{0.001}{10}$ h) $3.145 \times \frac{1}{10}$ i) $3.14 \times \frac{1}{100}$ j) 100×31.4 k) $10^2 \times 0.314$ l) $\frac{1}{100} \times 0.314$

Exercise 4: evaluate the following – express your answers in decimal format. Do not round off.

a) 3×0.1 b) $23. \times \frac{1}{100}$ c) $123 \times \frac{1}{1000}$ d) 1.23×10^2 e) $0.1 \times \frac{1}{10}$ f) $11 \times \frac{1}{100}$

g) 0.123×0.01 h) $0.007 \times \frac{1}{100}$ i) 700×0.01 j) 700×10^4 k) $700 \times \frac{1}{10000}$ l) $0.7 \times \frac{1}{10000}$

Convert numbers between whole numbers

In this section you will review and practice converting between three ways of displaying numbers between whole numbers (i.e. rates, decimals and per cents).

Summary of procedures
From rates to decimals: divide numerator by denominator

From rates to percents: convert to decimal then see below

From decimal to percent: multiply by 100 and slap on % sign

From decimal to rate: write out the decimal over the appropriate denominator and simplify (see example below)

From % to rate: % means per 100; write x% as rate x/100 and simplify

From % to decimal: write as rate per 100 and convert from rate to decimal.

From decimal to rate example

eg. $2.325 = \frac{2325}{1000}$ convert to mixed number

$= 2\frac{325}{1000}$ divide by $\frac{5}{5}$ to reduce

$= 2\frac{65}{200}$ divide by 5/5 once more

$= 2\frac{13}{40}$ which is in lowest terms

Exercise 1: Convert the following as stated:

a) 4% to decimal	b) 0.22 to %	c) $\frac{2}{13} to$ %
d) 21.5% to rate	e) $\frac{13}{27} to$ decimal	f) 0.75 to %
g) $\frac{9}{27} to$ %	h) 3.22 to %	i) 81% to decimal
j) 0.73 to %	k) $\frac{2762}{3654} to$ %	l) 117% to decimal
m) 0.1 % to decimal	n) 0.05 to %	o) $4\frac{3}{4} to$ %

Portions of numbers: Taking a percent or other type of portion of a number means taking a piece out of it. This turns out to involve a pretty straightforward calculation, but make sure to think about what you are doing with it.

Exercise 1: Evaluate the following (example 1/2 of 10 means taking a half of 10, this is easy to conceptualize as we are comfortable with ½ as a concept. Procedurally it is easy too: ½ of 10 is written as 0.5×10; which equals 5)

a. One half of 8 b. 2/5 of 20 c. 1/8 of 16

d. one quarter of 44 e. 1/10 of 8 f. 1/8 of 10

Exercise 2: Evaluate the following (remember that 10% of 32 = 10% × 32) :

a) 34% of 27	b) 27% of 34	c) 5% of 176
d) 145% of 145	e) 50% of 46	f) 22% of 44
g) 24% of 77.99	h) 15% of 24.32	i) 6% of 2,995.99

Multipy and Divide Numbers Between Whole Numbers when intuition may not work

If you know you are rusty with procedures with rates, make sure to look for the pdf booklet found at www.stataras.com.

Steps for multiplying rates:

1. convert to improper rates if needed
2. multiply numerators and multiply denominators
3. reduce to lowest terms

Example 1 :

$\frac{2}{5} \times \frac{3}{7} = \frac{2\times3}{5\times7}$

$= \frac{6}{35}$

Example 2 :

: $3\frac{3}{4} \times \frac{2}{3} = \frac{15}{4} \times \frac{2}{3}$

$= \frac{15\times2}{4\times3}$

$= \frac{30}{12} \Rightarrow$ this can be reduced

$= \frac{30\div6}{12\div6}$

$= \frac{5}{2} \Rightarrow$ change to mixed number $= 2\frac{1}{2}$

Steps for dividing rates:

1. convert to improper rate if needed
2. find reciprocal of divisor & change to mult'n
3. multiply numerators and multiply denominators
4. reduce to lowest terms

Example 3 :

$\frac{1}{6} \div \frac{3}{4} = \frac{1}{6} \times \frac{4}{3}$

$= \frac{4}{18} \Rightarrow$ this can be reduced

$= \frac{4\div2}{18\div2}$

$= \frac{2}{9}$

Example 4 :

$2 \div \frac{5}{12} = \frac{2}{1} \times \frac{12}{5}$

$= \frac{2\times12}{1\times5}$

$= \frac{24}{5} \Rightarrow$ reduce to mixed number $= 4\frac{4}{5}$

Exercise 1:

a) $\frac{1}{5} \times \frac{1}{7}$ b) $\frac{1}{3} \times \frac{6}{7}$ c) $5 \times \frac{1}{2}$

d) $\frac{3}{5} \div \frac{7}{8}$ e) $\frac{1}{6} \div \frac{1}{6}$ f) $3 / \frac{1}{12}$

g) $\frac{1}{2} \times \frac{1}{4}$ h) $\frac{3}{4} \times \frac{3}{4}$ i) $\frac{2}{5} \times \frac{3}{4}$

j) $2\frac{1}{2} \times \frac{2}{3}$ k) $5 \div \frac{5}{12}$ l) $\frac{5}{6} \times \frac{6}{7}$

Add and Subtract Numbers between whole numbers

You can only add and subtract rates that are like i.e. that have a common (the same) denominator.

$\frac{1}{2}$ and $\frac{3}{2}$ are like, $\frac{1}{2}$ and $\frac{2}{3}$ are **not** like

Exercise 1: Circle the pairs of rates, that are ***like***, and cross out those that are ***not like***.

a) $\frac{1}{3}$ $\frac{1}{2}$ b) $\frac{4}{5}$ $\frac{11}{5}$ c) $\frac{9}{2}$ $\frac{9}{5}$ d) $\frac{4}{5}$ $\frac{3}{10}$ e) $\frac{6}{7}$ $\frac{9}{7}$ f) $\frac{5}{6}$ $\frac{1}{6}$ g) $\frac{3}{8}$ $\frac{3}{7}$

Steps for addition and subtraction of rate***:***

1. convert to a/b format if needed
2. find common denominator if needed
3. add/subtract numerators and keep denominators the same
4. reduce to lowest terms

In examples 1 and 2 rates are like, thus there is no need to find a common denominator.

Example 1. $1\frac{3}{5} + \frac{1}{5} = \frac{8}{5} + \frac{1}{5}$

$= \frac{8+1}{5}$

$= \frac{9}{5}$

$= 1\frac{4}{5}$

Example 2: $\frac{1}{4} - \frac{3}{4} = \frac{1-3}{4}$

$= \frac{-2}{4}$ $\Rightarrow$ reduce

$= \frac{-1}{2}$

Exercise 2: Evaluate the following.

a) $\frac{1}{3} + \frac{1}{3}$ b) $\frac{7}{5} - \frac{4}{5}$ c) $\frac{9}{2} + \frac{9}{2}$

d) $\frac{4}{5} - \frac{3}{5}$ e) $\frac{6}{7} + \frac{9}{7}$ f) $\frac{5}{6} - \frac{1}{6}$

Add and Subtract Numbers between whole numbers (continued)

In the examples below rates don't have common denominators.

Example 3

$$\frac{1}{3} + \frac{2}{5} = \frac{1\times5}{3\times5} + \frac{2\times3}{5\times3}$$
$$= \frac{5}{15} + \frac{6}{15}$$
$$= \frac{5+6}{15}$$
$$= \frac{11}{15}$$

Example 4

$$\frac{1}{4} - \frac{2}{8} = \frac{1\times2}{4\times2} - \frac{2}{8}$$
$$= \frac{2}{8} - \frac{2}{8}$$
$$= \frac{2-2}{8}$$
$$= \frac{0}{8}$$
$$= 0$$

Example 5

$$3\frac{1}{5} + \frac{2}{7} = \frac{16}{5} + \frac{2}{7}$$
$$= \frac{16\times7}{5\times7} + \frac{2\times5}{7\times5}$$
$$= \frac{112}{35} + \frac{10}{35}$$
$$= \frac{122}{35}$$
$$= 3\frac{17}{35}$$

Exercise 3: Evaluate the following.

a) $\frac{2}{3} + \frac{4}{9}$

b) $\frac{3}{4} - \frac{1}{8}$

c) $1\frac{1}{2} + \frac{1}{6}$

d) $\frac{4}{5} - \frac{1}{2}$

e) $\frac{6}{7} + \frac{1}{10}$

f) $\frac{5}{6} - \frac{1}{2}$

g) $\frac{1}{3} + \frac{1}{6}$

h) $7\frac{2}{3} + \frac{1}{10}$

i) $\frac{22}{7} - \frac{1}{17}$

Rounding numbers big and small

Rounding helps us get rid of detail that we don't need.
eg. There are 6,845,367,448 people living on the earth today, or ≈ 7 billion after rounding.

Using ≈ tells us that we are approximating… rounding is one way of approximation.

Before rounding you should remind yourself of the place names of the digits;
6,145,367,448 represents: 6 billions, 1 hundred million, 4 ten millions, 5 millions, 3 hundred thousands, 6 ten thousands, 7 thousands, 4 hundreds, 4 tens, and 8 ones.
4 398.32578 represents: 4 thousands, 3 hundreds, 9 tens, 8 ones, 3 tenths, 2 hundredths, 5 thousandths, 7 ten thousandths and 8 hundred thousandths

Step 1. locate the place name of the digit
Step 2. look to the right of that digit, to see if it is 5 or higher.
Step 3. If it is 5 or higher add one to the original digit, if not leave it alone.
Step 4. Make all subsequent digits into zeroes. You can truncate zeroes that are after the decimal sign.

Example:
Round 6,145,367,448 to the nearest billion ≈ 6 billion
Step 1. the 6 is the billions digit
Step 2. to the right is a 1, it is less than 5
Step 3. do not add one to the 6
Step 4. 6, 145, 457, 448 ≈ 6,000,000,000

Exercise 1:

a) round 345,679 to the nearest thousand

b) round 345.654 to 2 decimal places

c) round 399.6 to the nearest unit (one)

d) round 1,003,549 to the nearest hundred

e) round 3.003467 to the nearest hundred thousandth

f) round 34.75499 to 2 decimal places

g) round 34.75499 to the nearest thousandth

Always round off at the end of a series of calculations, not at the beginning or middle

Integer Addition and Subtraction Review

The Integers is the set of whole numbers and their negatives. −34 is an integer and 0.23 is not. This worksheet is designed to help you remind yourself of arithmetic with integers. If you are having difficulty with the questions below make sure to get an Integers lesson booklet from the math1112 page on www.stataras.com.

Exercises:

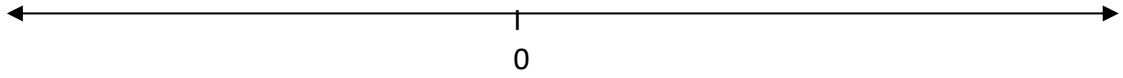
1. Rewrite the following phrases as integers.

   a) 15 metres below sea level ____ b) a profit of $ 117 ____

   c) 40 years younger ____ d) 25° C above zero ____

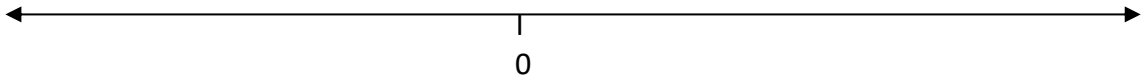
2. Evaluate the following using the number line as a guide.

   a) –4 + 3 b) 3 – 7 c) 17 - 5 d) –2 – 1



3. Evaluate the following using a number line.

   a) 7 + (+3) b) (–5) + (+7) c) (–3) – (–4) d) – 2 – 3



4. Evaluate the following:

   a) 7 + (+8) b) (–3) + (+4) c) (–2) + (–3)

   d) (+9) + (–3) e) (–2) + 5 f) –5 + 6

   g) 4 + (–3) + (–5) h) 42 – 23 – 19 i) 19 + 23 – 42

   j) 5 + 7 + (–3) k) 12 – 3 – (+4) – 1 l) 0 + 0 – 0 – (–0)

Integer Multiplication and Division Review

When multiplying integers, you don't just multiply the numbers you must also multiply the signs. Division works the same way. Remember that 9/3 is the same thing as 9 ÷ 3.

THE FOUR CASES

CASE 1 - first number positive, second number positive ⇒ answer is positive
CASE 2 - first number positive. second number negative ⇒ answer is negative
CASE 3 - first number negative, second number positive ⇒ answer is negative
CASE 4 - first number negative, second number negative ⇒ answer is positive

Exercises:

1. Evaluate the following

a) (+3)(+4)

b) (–2)(–12)

c) (+7)(–1)

d) 2(–4)(–3)

e) (+4)(–3)(–2)

f) (+3)(–8)(–2)

g) (+4) (–9)

h) 25(4)

i) (–32) (5)

j) (–5)(12)

k) 44 / (11)

l) 9 / 3

m) –4 / (–1)

n) –12/ (–2)

o) 10 / 2

p) 12 /6

q) –100 / 4

r) 34 / (–17)

s) –36 / 9

t) (–15) / (–3)

Exponents/Powers

Powers are a really good example of a short cut. We can replace $5\times5\times5\times5$ with 5^4.

Before going on it is a good idea to learn the terminology:

this is called the ***base*** $\rightarrow 5^4 \leftarrow$ *this is called the* ***exponent***

Definition: 5^4 is read as " 5 *to the exponent* 4" (or '5 to the 4th power')

Example 1: evaluate 9^2
Solution: $9^2 = 9 \times 9$
$= 81$

Example 2: $(-5)^3 = (-5)(-5)(-5)$
$= 25(-5)$
$= -125$

If you aren't sure how to multiply negative numbers make sure to review the Integers section

Special Case 1: any number *to the exponent* 1 stays the same examples: $5^1 = 5$, $8^1 = 8$, $10^1 = 10$, $a^1 = a$

Special Case 2: any number to the exponent 0 gets simplified to a 1 examples: $5^0 = 1$, $8^0 = 1$, $10^0 = 1$, $(blah)^0 = 1$

Exercise 1: evaluate all of the following powers

a) 6^2 b) $(-4)^3$ c) 2^3 d) 5^1 e) 3^5

f) $(-3)^1$ g) 9^2 h) 2^9 i) $(-2)^9$ j) $(-3478)^0$

Negative Exponents work a bit differently since negative here means reciprocal:

Examples $5^{-3} = \frac{1}{5\times5\times5}$ $10^{-1} = \frac{1}{10}$ $b^{-2} = \frac{1}{b\times b}$ *or* $\frac{1}{b^2}$

$= \frac{1}{125}$

Exercise 2: evaluate the following

a) 2^3 b) 2^{-3} c) 5^2 d) 5^{-2} e) $(3)^{-4}$

f) $(-3)^3$ g) 10^3 h) 10^{-3} i) 345^0 j) $(12 \times 73)^0$

Scientific notation:

Exercise 1: read over the table below. Fill in the blanks. It will help you with some of the conversion skills you need for you work with scientific notation.

Number in english	metric	Decimal form	Rate(a/b) form	As power of 10
One hundred	hecto	100	100	10^{2}
One hundredth	centi	0.01	1/100	10^{-2}
One million	mega	1 000 000	1 000 000	10^{6}
One millionth	micro	0.000001	1/1 000 000	10^{-6}
One thousand	kilo			
One thousandth	milli			
Ten thousand	n/a			
Ten thousandth	n/a			
Hundred thousand	n/a			
Hundred thousandth	n/a			
Ten	deca			
One tenth	deci			
Three tenths	deci	0.3	3/10	3.0×10^{-1}

Scientific Notation is essentially a shorter way to write either very large numbers or very small numbers. You can see how in the above situations the powers relate to the decimal form of simple numbers.

Exercise 2: Convert the following from scientific notation (simply evaluate the expression)

a) 3.1415×10^{3}

b) 0.005×10^{5}

c) 344.12×10^{-2}

d) 0.1453×10^{-1}

e) 799.0033×10^{6}

f) 3.0×10^{-3}

Exercise 3: Convert the following to scientific notation (start with ____ $\times 10^{??}$)

a) 3,446,112

b) 0.005

c) 13,443

d) 1,453,887

e) 799

f) 0.0000432