Math 1112

Unit 6 Set-ting Objects in Order

6.0: Introduction to Sets as the logic of belonging, and set relations

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Author:

Sets: ***An Introduction to the logic of belonging***

Terminology and notation

A ***set*** is a collection of 'things' which are called ***elements***.
There are two ways of representing sets.
1. describe the collection by explaining what makes the elements a group,
e.g. A = The set of all positive odd numbers less than 10
2. make a list and enclose the elements in brackets – called ***roster*** method.
e.g. A = {1,3,5,7,9}.
A ***well-defined set*** is one in which it is clear who is in and who is not – a set that is not well defined is not really a set at all

The ***cardinality*** of a set is the number of elements in it. The symbol for the cardinality of an infinite set is $\aleph_0$, and for a null set is {} or Ø

Two sets are ***equivalent*** if they have the same cardinality, this is also called ***one to one correspondence:*** two sets are in one-to-one correspondence if each element in the first can find a 'pair' in the second set.

Two sets are ***equal*** if they contain exactly the same elements. {1,3,5,7,9} is equivalent to {2,4,6,8,10} but not equal. Order does not matter thus {$, %, ^} is equal to {^, %, $}.

Finite sets have a finite # of elements while ***infinite sets*** have an infinite # of elements
eg. {1, 2, 3} is finite, eg. {1, 2, 3…} is infinite

<u>Exercise 1:</u> what is the cardinality of the following sets. Use set notation for infinite and null sets.

A = Canadian provinces and territories

B = Black markers that are green

C ={a,b,c,d,e,f,g,h,i,j,k,l,m}

D = Positive factors of 96

E = { 2,4,6,8,10……}

F = distinct letters in the word 'prime'

G = Politicians

H = {1,4,7,10, …34,37}

I = vowels in 'eunoia'

J = numbers found between 3 and 4

<u>Exercise 2:</u> Which of the above pairs of sets are equal?

<u>Exercise 3:</u> Which of the above pairs of sets are equivalent?

<u>Exercise 4:</u> Which of the above sets are infinite?

Listing sets practice

Exercise 1: List the elements of the following sets and find the cardinality of it. If you believe that the set is not well-defined simply state ‘not well-defined’ and move on.

a. One die is rolled and you are interested in the side facing up. List the set of all possibilities. How many distinct possibilities are there?

b. Two coins are tossed at the same time (a nickel and a penny). List the set of all possible outcomes for side facing up. How many distinct possibilities are there?

c. There are two marbles in a box, but you don’t know their colours. The only thing you are told is that ***if*** one is a red marble, ***then*** the other is a blue marble. List the set of all possibilities. How many distinct possibilities are there?

d. You are purchasing two baby geckos and really want to get a male and female, but it is nearly impossible to know their sex until they grow into adulthood. List the set of all possible outcomes for the sex of the two geckos. How many distinct possibilities are there?

Listing sets practice

e. You have a bag containing 7 marbles numbered from 1 to 7. The first 3 marbles are green, next, 3 are red and the last one is blue. List the set of all possible outcomes when you reach in and choose one marble. How many distinct possibilities are there?

f. Three coins are tossed at the same time (a quarter, a nickel and a penny). List the set of all possible outcomes for the sides facing up. How many distinct possibilities are there?

g. 2 six sided dice are rolled and of interest are the numbers facing up. List the set of all possible outcomes for the side facing up. How many distinct possibilities are there?

h. List the set that contains all prime numbers less than 41. How many elements does it contain? (1 is not a prime number so don't include it😊)

Relations between sets

Terminology: Let's start by defining a few sets for example purposes.

U = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,17,34,51} A = {1,3,5,7,9,11,13,17}

B = {3,5,7} C = {1,2,3,4,5} D = {8,10,12}

E = positive factors of 10 that are >50 F = positive even numbers that are <14

universal set: contains all elements under consideration in a given discussion.
Set U will be treated as the universal set, but its contents can change from question to question.

empty set (or null set): contains no elements
E is an empty set { }, or null set since there are no positive factors of 10 that are > 50.

When examining two sets they can be related in one of 3 different ways. (see diagrams next page)

Subset ($\subseteq$) : Set X is a subset Y if all elements of set X are also in set Y.
I*n examples above* $B \subseteq A$, and all are subsets of U (E is a null set, thus a subset of all other sets)

proper subset (***$\subset$***): a subset that does not contain all elements of the original.
A, is a proper subset of U; i.*e.* $A \subset U$**,**

disjoint sets: contain no elements in common. Also called mutually exclusive. *A is disjoint with D.*

overlapping (intersecting) sets: 2 sets in which each contains some, but not all, elements in common with the other. *C and A are overlapping sets*.

Make sure to study set relations on next page carefully before doing exercise 1.

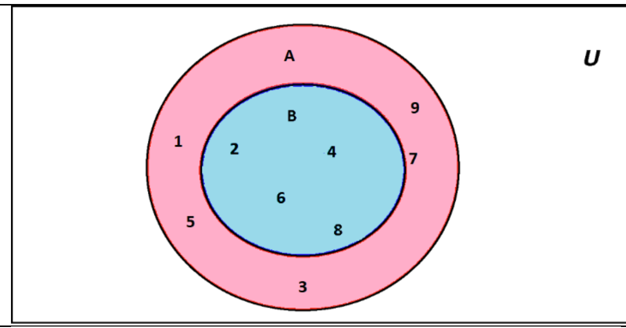
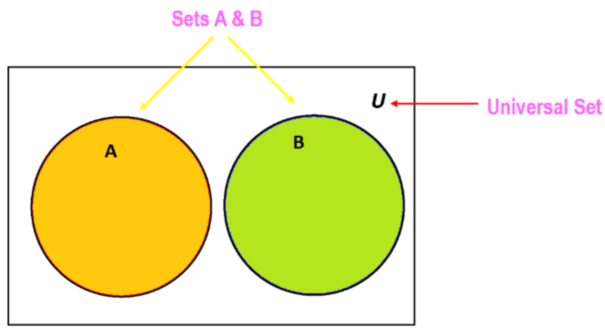
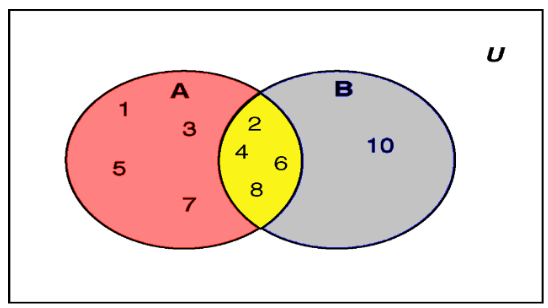
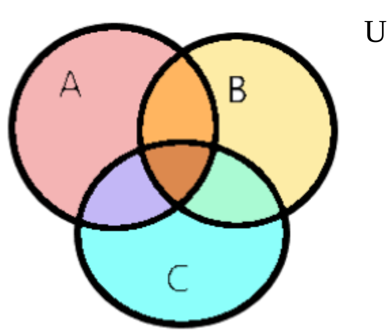
Exercise 1: Given U = {1,2,3,4,5,6,7,8,9,10,11,12,14,17,34,51}
A = {1,3,5,7,9,11,13,17}, B = {3,5,7} C = {1,2,3,4,5}
D = {8,10,12} E = positive factors of 10 that are >50
F = positive even numbers that are <14

Describe the relationship between the following pairs of sets as either disjoint, overlapping or subset (e.g. X & Y are _?????_ sets). sketch the relation for practice

a. A & B are	b. A & C are	c. A & D are

d. B & C are	e. B & D are	f. D & F are

Relationship of Sets visualized as Venn diagrams
Given: U = the set of all whole numbers, 2 sets A and B could be related to each other as follows.

Subset If **A = { 1,2,3,4,5,6,7,8,9}** and **B = { 2,4,6,8}** then **B** is a subset of **A** **$B \subseteq A$ or written 'backwards' $A \supseteq B$** Note that B is also a proper subset of A i.e. $B \subset A$	
Disjoint set **If A = { 1,3,5,7,9} and B = { 2,4,6,8}** Then they have no elements in common and we say that **A and B are disjoint sets** **$A \not\subset B$; $B \not\subset A$**	
Overlapping sets **If A = { 1,2,3,4,5,6,7,8,9} and** **B = { 2,4,6,8,10}** We see that elements {2,4,6,8} are common to both A and B, thus the sets are overlapping.	
Visual of 3 overlapping sets A, B, C Some elements are common to all 3 sets, some are common to two sets, and some are only in one.	

Set operations: we will look at three basic operations – union, intersection and complement.
Let's start by setting up a scenario first:

Example 1:

U – will act as the universal set defined as the whole numbers U = {1,2,3,4,5,6,7,…….}
A – defined as the set of whole numbers ≤ 8; i.e. A = {1,2,3,4,5,6,7,8 }
B – defined as the set of even whole numbers ≤ 10; i.e. B = {2,4,6,8,10}

We can see that A and B are overlapping sets (they have 2,4,6,8 in common. This is an important first step in operations with sets as the result of operations is dependent on the initial relation.

Union (∪): create a set of all elements in both sets. Make sure not to list any elements twice.
$A \cup B$ = {1,2,3,4,5,6,7,8,10}

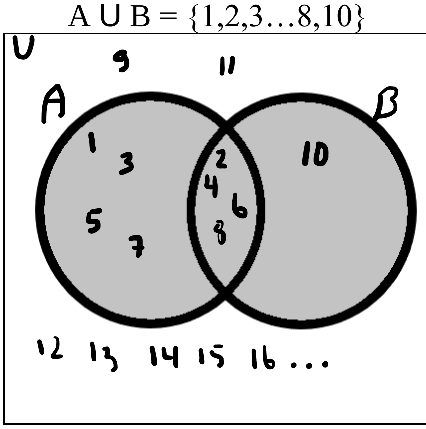
Intersection (∩): create a set of all elements common to the two (or more) overlapping sets. If the sets are not overlapping then the intersection is the null set {};
$A \cap B$ = {2,4,6,8}

Complement: ($A^{complement}$ is written as $\bar{A}$ or A^C) the set of all elements in the universal set, but not in the named set.
A^C= {9,10,11,12,13,14,15,16……}

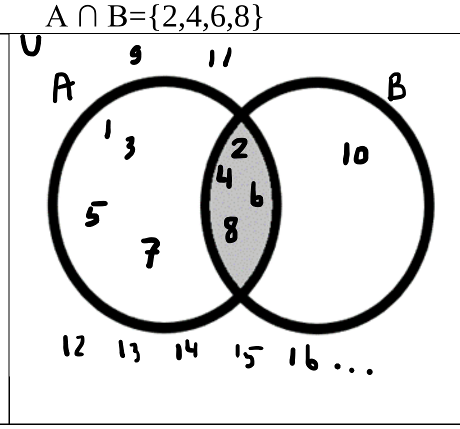
Visualizing operations: (shading)

The above operations on A and B can be described visually through shading. The examples below show the results of the operations as shaded shapes. I have not included the steps that help with more complex combinations of operations.

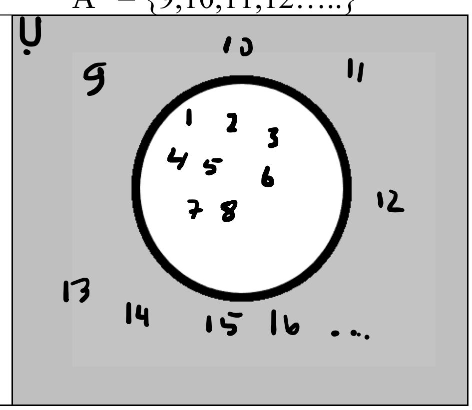
$A \cup B$ = {1,2,3…8,10}



$A \cap B$={2,4,6,8}



A^C = {9,10,11,12…..}



Exercise 1: Given U = {1,2,3,4,5,6,7,8,9,10,11,12,13} A = {1,3,5,7,9,11,13}, B = {3,5,7}
C = {1,2,3,4,5} and D = {8,10,12}

Draw/shade a sketch for each question and list the elements of the resulting set. I have set up tables so that you can do it in three steps: First draw the correct relation and shade first region, then shade second region – and finally combine them appropriately. In some cases you may only need 2 steps.

a) $A \cup B$	b) $A \cap C$	c) $A^C \cup C^C$	d) $(A \cap B)^C$

e) $(A^C \cap C^C)^C$	f) $A^C \cap C$	g) $C \cap D$	h) $(C \cup D)^C$

Exercise 2: Quick review of set theory shading exercises

Given that U = { 0,1,2,3,4,5,6,7,8....}; A = {1,3,5,7,9}; B = {2,4,6} and C = {1,2,3,4,9}

Draw appropriate Venn diagrams and shade in the following regions. (I have set up tables so that you can do it in three steps)

First draw the correct Venn diagram and shade first region, then shade second region – and finally combine them appropriately. In some cases you may only need 2 regions.

a) $A^C \cap C$	b) $A \cap B^C$	c) $A^C \cap B^C$	d) $(A \cup C)^C$

e) $(A \cap B)^C$	f) $A \cup C^C$	g) $A^C \cap B^C$	h) $(A \cup B)^C$

Taking subsets of a set

Every set can have a multiple of subsets of various sizes, with the smallest being the null set and the largest being the original set. Make sure to review the definitions of subset from page 3.

Weird aspect of subsets to remember: the null set is a subset of every set. Thus the set {1} has 2 subsets, the set {1} and {}.

Example 1: Find all subsets of the set B = {2,4,6,8}. How many subsets are there of set B?

Solution: The solution comes from a meticulous search for all possible pairings (subsets of size 2 and 3 are a particular challeng) from B to get:

{}						null set;
{2}	{4}	{6}	{8}			cardinality = 1
{2,4}	{2,6}	{2,8}	{4,6}	{4,8}	{6,8}	cardinality = 2;
{2,4,6}	{2,4,8}	{2,6,8}	{4,6,8}			cardinality = 3;
{2,4,6,8}						cardinality = 4, and the original set.

There are 16 subsets of set B.

<u>Exercise 1a:</u> Find all subsets of Q = {2,4,6}

1b: How many subsets of Q are there? _______

<u>Exercise 2a:</u> Find all subsets of T = {3,6,9,12}

2b: how many subsets of T are there? _______

<u>Exercise 3:</u> Use your answers to 1b and 2b to come up with a pattern for the number of subsets of a set related to the cardinality of the original set. Use it to predict how many subsets a set with cardinality = 5 will have.

Set operations applied to concrete world:

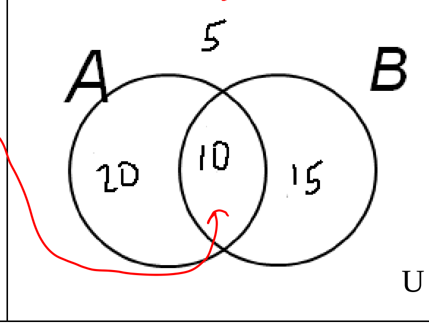
In these sketches the numbers represent cardinality i.e # of individuals

***Example 1**:* In a class of 50 first year HIM students, 30 are good at math (A), 25 are good at science(B), and 10 are good at both.

How many first year HIM students are good at both? 10

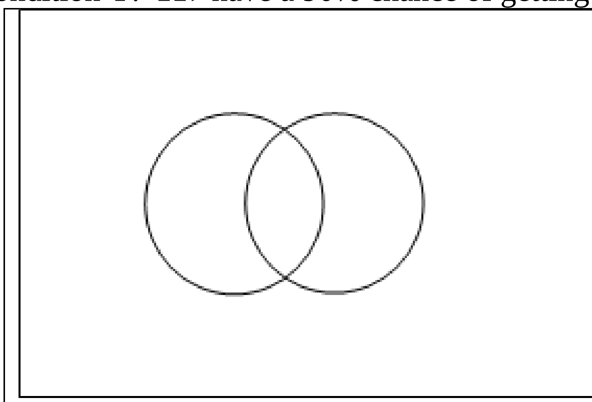
How many are not good at either? 5

The diagram to the right makes it much easier to get the correct answer.



Practice: Label shade in the Venn diagram below given the following scenarios, after answering the given questions.

Exercise 1: 500 residents of a city are tested for the presence of condition X, and Y. If they have both then have a 90% chance of getting disease Q. 320 have condition X, 272 have condition Y. 117 have a 90% chance of getting disease Q.



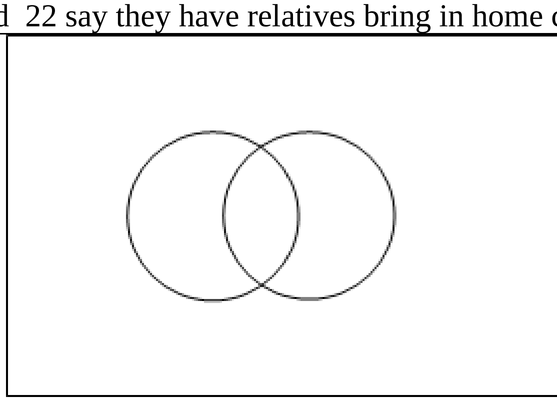
a) How many residents have both condition X and condition Y?

b) How many residents have neither condition X, nor condition Y?

c) How many only have condition X?

d) How many only have condition Y?

Exercise 2: 48 patients in a hospital are asked what food they eat. 35 say they eat hospital food, and 22 say they have relatives bring in home cooked food. 14 say they eat a bit of both.



a) How many residents eat both hospital and home cooked food?

b) How many residents eat neither hospital nor home cooked food?

c) How eat only hospital food?

d) How many only eat home cooked food?

Bonus exercise: for fun – try to set up these problems as contingency tables. The problems are solvable using both methods.