Math 1112

Unit 3: Algebra with lines and curves

The Cartesian Coordinate System

The Cartesian coordinate system is named after the 17th century French mathematician, philosopher, theologian and designer of cannons, Rene Descartes, but also attributed to Pierre de Fermat, a civil servant, whose duties among other things was to sentence priests to death by burning during the Plague, and the Inquisition. Who said mathematics history is boring☺

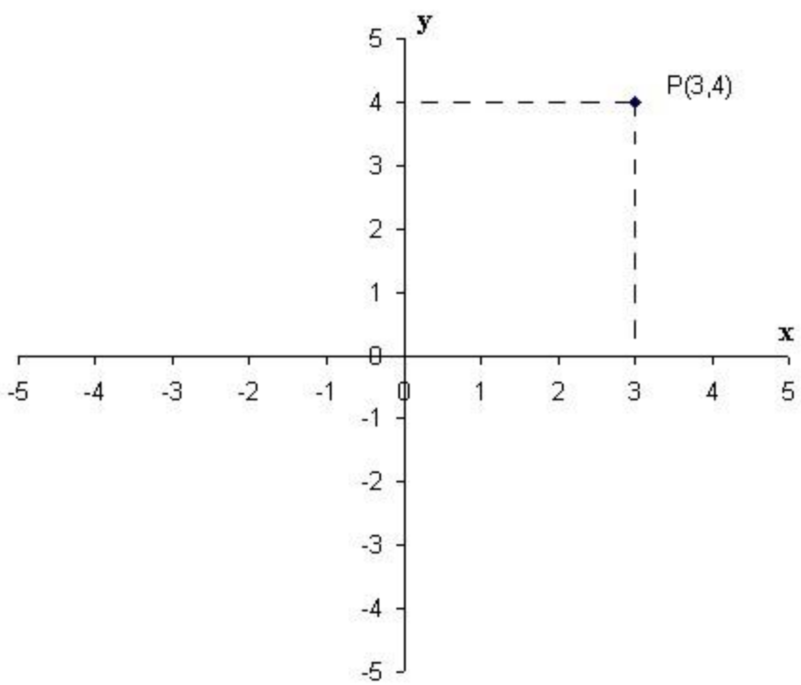
The basic idea comes from the goals of algebra – to describe the world through mathematics. The Greeks did a good job with geometry, but were limited in their work with algebra. After Descartes and others, mathematicians could transform geometric problems into algebraic equations and solve many problems that the Greeks could not.

What is the Cartesian Coordinate system? It is a way of mapping any flat surface and allows one to orient any object on that surface in relation to a central point (0,0).

The Cartesian coordinate system is a direct extension of the number line with a central 0. Instead of a single number telling us how far we are from 0, with the coordinate system we will have two numbers (x,y) (x-axis is horizontal, y-axis is vertical)

x – tells us how far we are to the right (+) or left (-)

y – tells us how far we are above (+), or below (-)



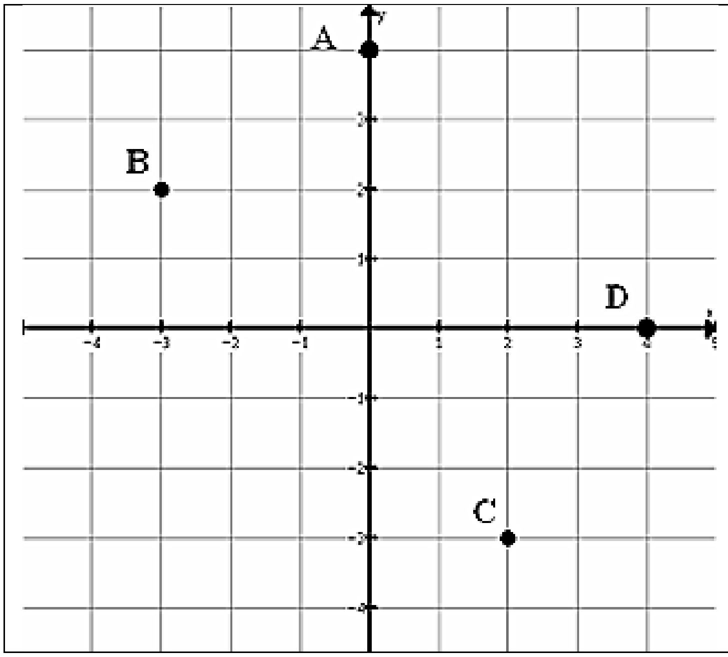
Point P (3,4) is located 3 units to the right of 0 and 4 units above 0.

P can be called a point, an ordered pair or a coordinate.

Exercise: Add in point Q(-4,3) R(-3,-4) and the dashed lines to indicate distances from (0,0)

Practice with Ordered pairs.

<u>Exercise 1:</u> Write the ordered pairs for each of the points below



A (,)

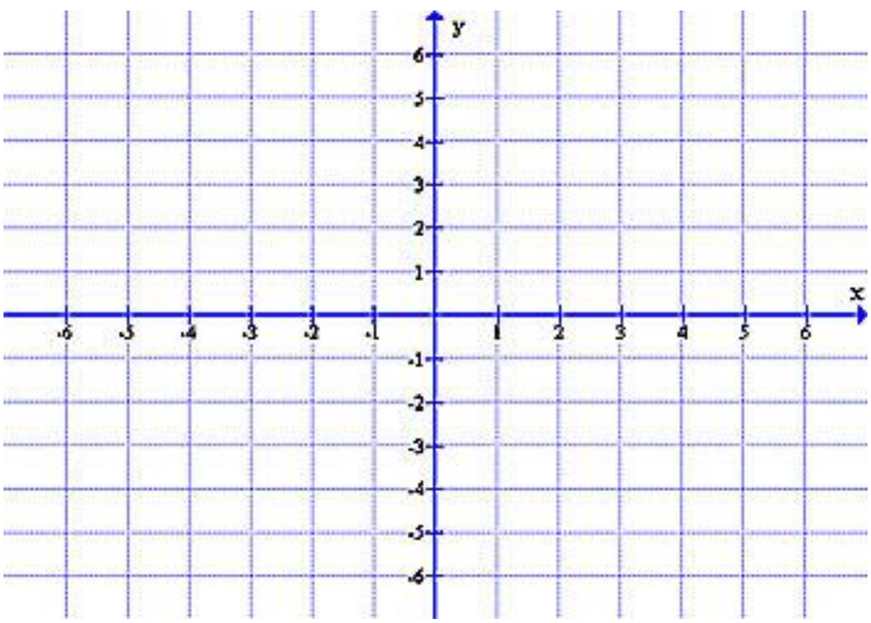
B (,)

C (,)

D (,)

***<u>Exercise 2</u>*:** Plot the following ordered pairs on the graph below

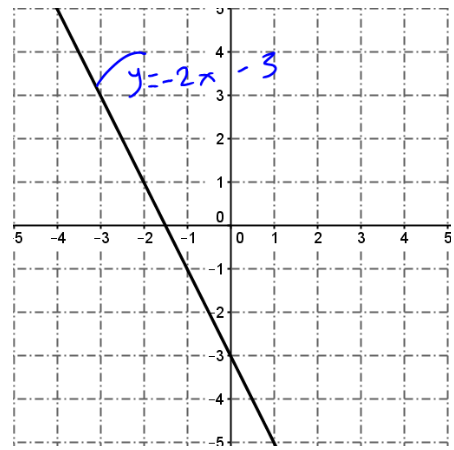
A(-5, 7) B(- 1.5, 0) C(1,-5) D(-5,-3) E (0,5) F(4,3)



The straight line as a set of points defined by a linear equation

Look back at ***Exercise 2*** on the previous page and note that there is a relationship between the points A, B,& C: they are in a line. The algebraic representation (an equation with 2 variables) of the line that joins the three points (A, B, C) is $y = -2x - 3$.

A ***line*** is a relation comprised of a series of an infinite number of points that can be represented geometrically or algebraically as a linear equation.



Linear equation:
standard form: $ax+by+c=0$ e.g. $-2x - y - 3 = 0$
slope, y-intercept form : $y = mx+b$ e.g. $y = -2x - 3$

Two types of problems you could confront in this section:
1. Given an algebraic, equation plot a graph.
2. Given a graph find the slope and y-intercept.

Steps to happy plotting

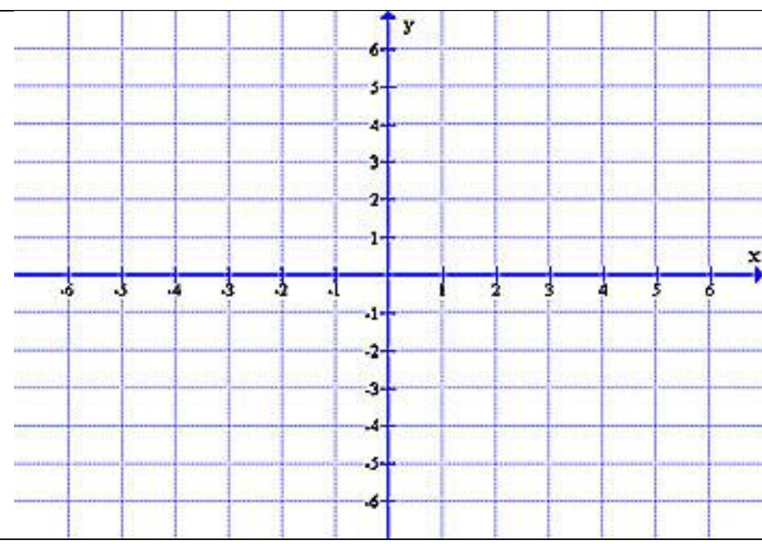
Step 1: set up a blank table of values.
Step 2: choose any 3 ‘x’ values that aren’t too large.
Step 3: substitute each of the ‘x’ values into the equation to generate a related ‘y’ value
Step 4: write out the ordered pairs
Step 5: plot each of the points on a graph.
Step 6: join the points to make a line.

<u>Exercise 1:</u> follow the six steps to get a plot for $y = 3x - 2$

table of values

x	y

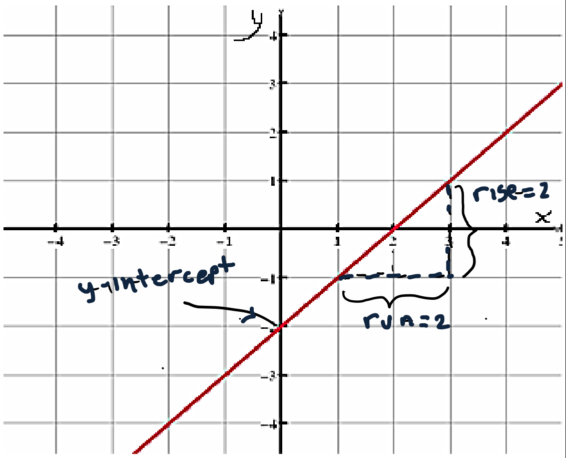
list of ordered pairs



Something to think about: The line is a set of points that is a set of solutions to the equation $y = 3x-2$. How many solutions (values of x and y) are there?

Slope and y-intercept – example and practice

The graph below is a visual representation of $y = x - 2$;



Every graph of a line including the graph to the left can be written as a linear equation in the form **y=mx+b**.
In this format the equation highlights 2 features of lines.

Slope (m) is a measure of how steep a line is (larger number steeper slope) and whether it is ascending (climbing) left to right (+ slope) or ascending (climbing) right to left.(–slope)

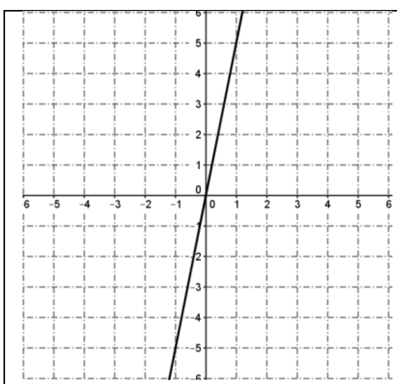
The slope can be found by calculating $m = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$ between two points.

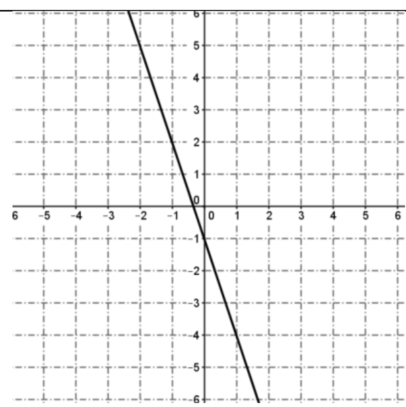
y-intercept (0,b) tells us where the line crosses the y-axis Try to calculate the slope (m) and find the y-intercept for the line to the left.

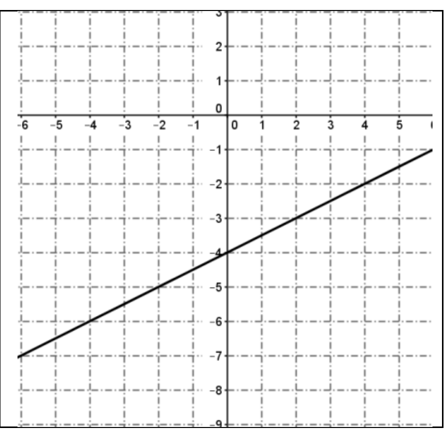
$\frac{rise}{run} = \frac{2}{2} = 1$ slope: m = 1, y-intercept: b = -2 or (0, -2)

Exercise 1: Find the slope and y-intercept for the following:

a. m = b = b. m = b = c. m = b =







notes: horizontal lines have slope: m = 0 and equation y = c
vertical lines have undefined slope (since run = 0)
and equation x = c.

Practice with Graphing lines

<u>Exercise 1:</u> For each of the equations below follow the six steps to plot a line, then write out the slope and the y-intercept.

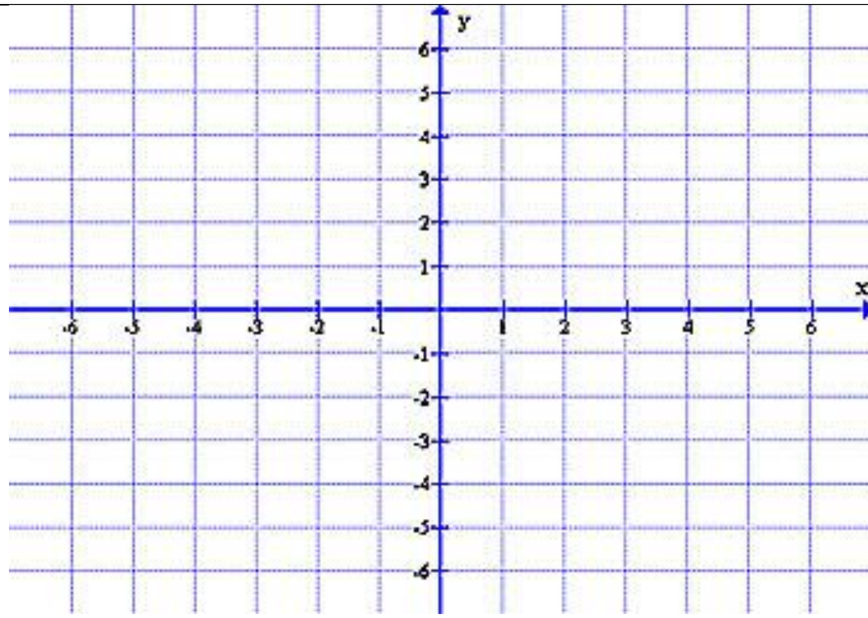
a) y= –x+6

table of values

x	y

list of ordered pairs

m =
b =



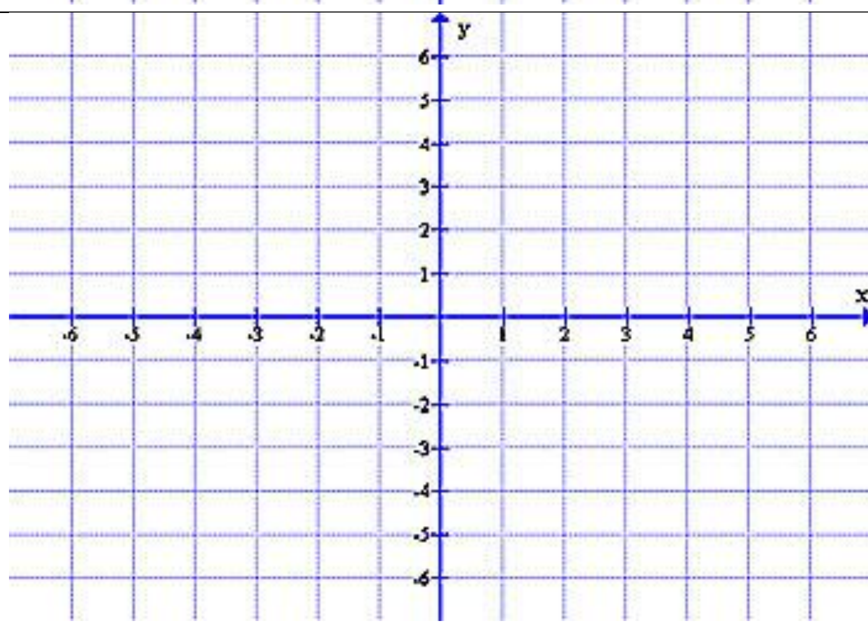
b) y = 3x – 2

table of values

x	y

list of ordered pairs

m =
b =



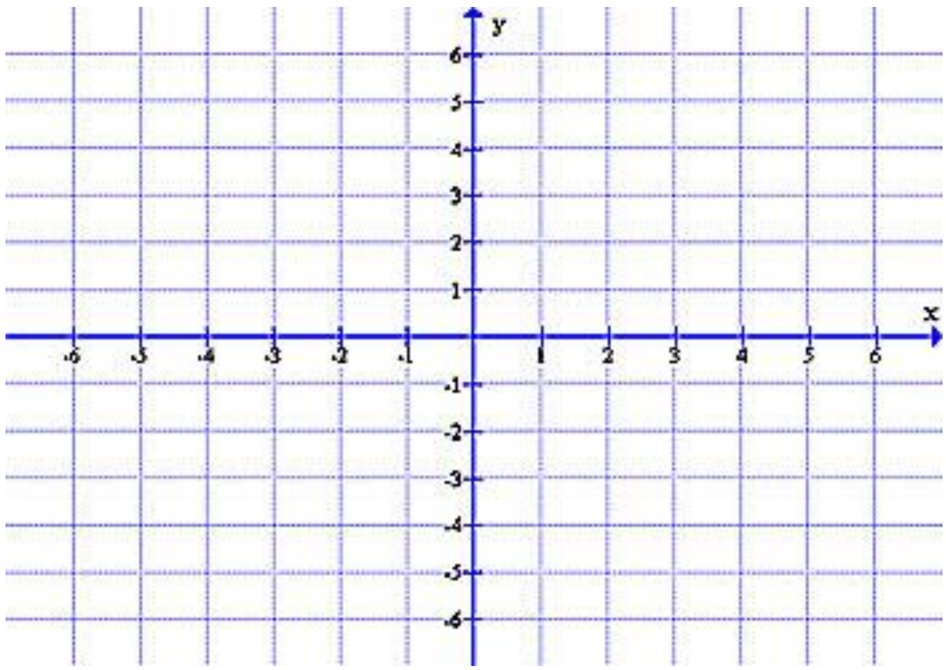
for c) and d) do work below and plot them on the graph paper above or on blanks at back of book.

c) y = 4x+5 d)y = -3x +5

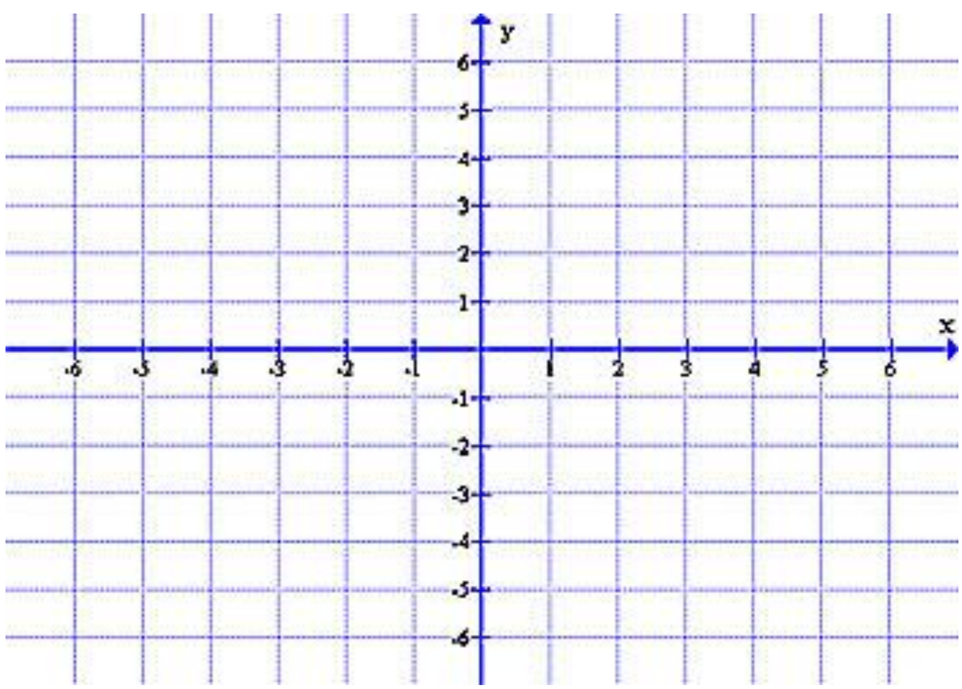
Practice with Graphing lines – continued

Exercise 2: Use your skills in solving equations to rewrite the following in y = mx + b form, then graph them below (don't forget to label them using y=mx+b form)

a) $x = y - 4$ b) $x = y + 5$ c) $3x + y = 6$



d) $2x - 2y = 2$ e) $x = y/4 + 1/2$ h) $3x = 2y$



Finding the point of intersection of 2 lines algebraically ***(solve simple sets of 2 equations)***

Solving for two unknowns (x,y) with two simple equations using substitutions

This section brings together many skills. You will have to solve for 2 unknowns, but will be given 2 equations.

Leftside Right side

i.e. $y = m_1x + b_1$

$y = m_2x + b_2$

Steps to solving simple sets of 2 equations:

Step 1: Write both equations in y = mx+b format

Step 2: Since y = y then the mx + b parts must be equal too. The right sides of both equations are equal and we can write out the resulting equation now with only one variable 'x'

i.e. $m_1x + b_1 = m_2x + b_2$

Step 3: solve for x

Step 4: Substitute the value of x into one of the original equations of the line to solve for y

Step 5: your solution is (x,y)

Example 1: Find the value of x and y from the following equations

I. y = 3x + 2

II. y = 2x + 7

} both are in y = mx + b format.

Step 1: Equations are both in y=mx+b format.

L.S. R.S.

y = 3x + 2, and

y = 2x + 7,

– the Leftsides of both equations are equal

Step 2: since y = y then it must be true that 3x + 2 = 2x + 7.

∴ the Right sides must be equal too

{*we have one equation with one unknown*}

Step 3: 3x + 2 – 2 = 2x + 7 – 2 {*subtract 2 from both sides to isolate x on the left side*}

3x = 2x + 5 {*now subtract '2x' from both sides*}

3x – 2x = 2x + 5 – 2x

x = 5 {aha… we have a value for x, but not yet for y}.

Step 4: {*Substitute x = 5 into either of the original equations to get the value of y*}

substituting x = 5 into y = 3x + 2 y = 3(5) + 2 y = 15 + 2 y = 17	Or substituting x = 5 into y =2x + 7 y = 2(5) + 7 y = 10 + 7 y = 17 {*the same answer as to the left. The answer must be the same, otherwise you have made an error somewhere.*}
Solution x = 5; y = 17 written (5, 17)	Solution: x = 5; y = 17 written as (5, 17)

Step 5: The point of intersection of the two lines is (5,17)

Finding the point of intersection of 2 lines cont'd

Example 2: Find the point of intersection of the following two lines.

I. y = 3x + 2

II. x + y = 7 *{Take a good look at the two equations.}*

Step 1. Equation I is in y=mx+b format, but equation II is not.

I. y = 3x + 2,   II x + y = 7 {I want to isolate the y so will subtract x from both sides}

x – x + y = 7 – x

y = 7 – x

Step 2. Since y = y then it must be true that 3x + 2 = 7 – x (one equation with one unknown)

Step 3. 3x+2 = 7 – x {subtact 2 from both sides}

3x+2 – 2 = 7 – 2 – x

3x = 5 – x {add x to both sides to make sure that x is only on one side of the equation}

3x + x = 5

$\frac{4x}{4} = \frac{5}{4}$ {divide both sides by 4}

x = 1.25 *{ we have a value for x, but not yet for y.}*

Step 4. {Substitute the value of x into either of the equations to get the value of y}

substituting x = 1.25 into y = 3x + 2 y = 3(1.25) + 2 y = 3.75 + 2 y = 5.75	substituting x = 1.25 into x + y = 7 1.25 + y = 7 *{subtract 1.25 from both sides}* 1.25 – 1.25 + y = 7 – 1.25 y = 7 – 1.25 y = 5.75 {the same answer as to the left. The answer must be the same, otherwise you have made an error somewhere.}
Solution x = 1.25; y = 5.75	Solution: x = 1.25; y = 5.75

Step 5. The point of intersection of the two lines is (1,25, 5.75)

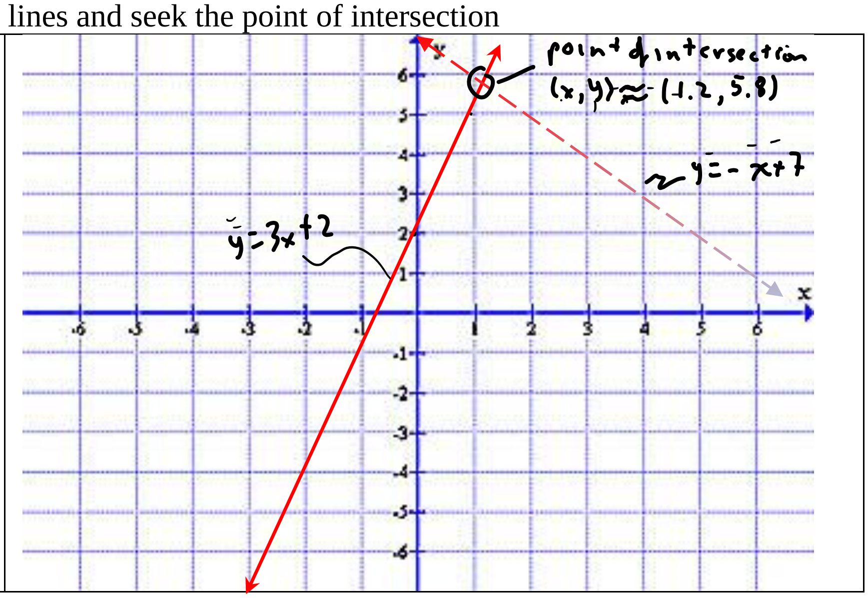
Visual solution: plot both lines and seek the point of intersection

I. y = 3x + 2

II. x + y = 7 i.e. (y=-x+7)

Find points for each line. Then plot them and join the points to produce two lines.

the point of intersection will need to be estimated.



Find point of intersection of 2 lines practice

Exercise 1: Solve the following systems of equations algebraically. (i.e. find the point of intersection) You can plot the lines on graph paper if you wish for practice.

a. $y=x+1, \quad y=-x+3$

b. $y=x+1, \quad y=-x+5$

c. $y=2x-4, \quad y=-2x+8$

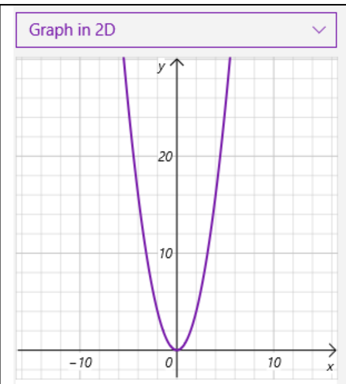
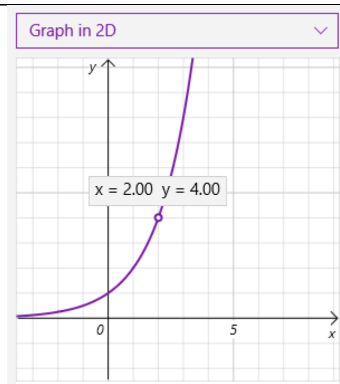
d. $3x-y=1, \quad 2x+1=y$

e. $y+4x=10, \quad 2x-2=y$

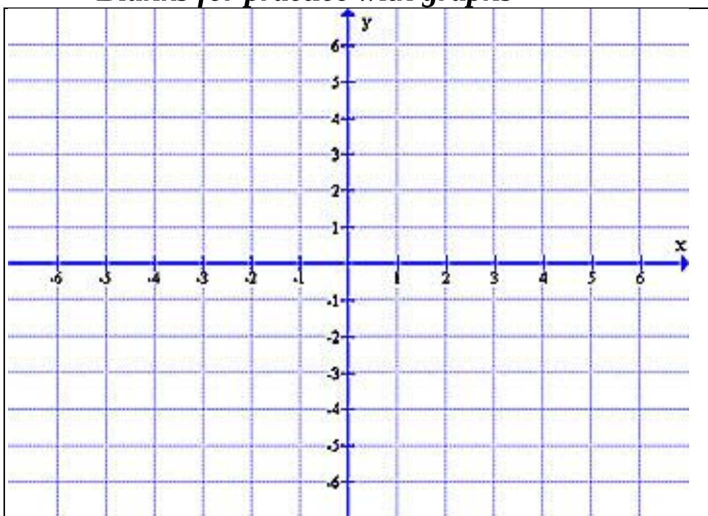
f. $2y+x=4, \quad y=7+2x$

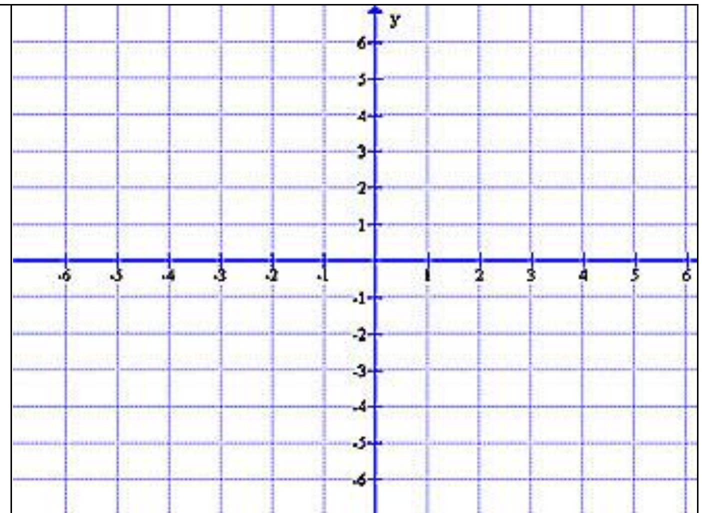
Characteristics of non-linear equations

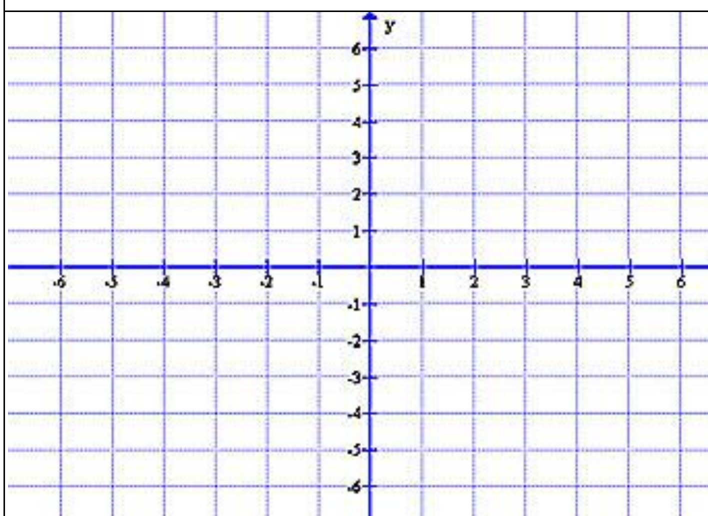
Many types of non-linear equations, related to many types of 'curves' have been identified by mathematicians. (parabola, exponential function, ellipse, circle, hyperbola are ones that you may have come across in your studies. We will take a peak at two of them, the parabola and the exponential function. (parabola is pretty simple to think about and with, and exponential has been very useful in contributing to the understanding of distribution of disease and over time.)

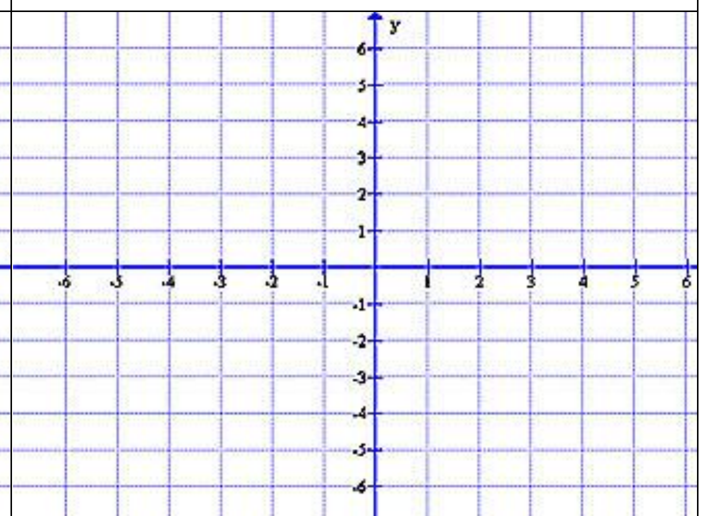
Parabola: typical format of equation with vertex at (0,0) is $y=ax^2$; the graph to the right shows a visual representation where a = +1. (positive means opening up, when value of avalue of a increases, the graph goes 'up faster', i.e. is more narrow. A more complex looking algebraic representation for a graph with vertex at (h,k) is $y = a(x-h)^2+k$	
Exponential function: typical format of equation is $y=b^x$; note that the variable x is the exponent. b indicates the speed of growth of the function. The graph to the right represents $y = 2^x$. Every additional whole number step for x means a doubling of y. Note that b >0; as otherwise there is no continuous curve joining the points. Also, for b = 1, the curve becomes the line y = 1.	
What happens to $y=b^x$ when 0< b < 1; or when b is between zero and 1. Plot $y = 0.5^x$, now try other values of b below 1 and above zero to find out using OneNote or geogebra	
For fun: plot the following exponential equation $y = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ in OneNote or using geogebra/desmos	

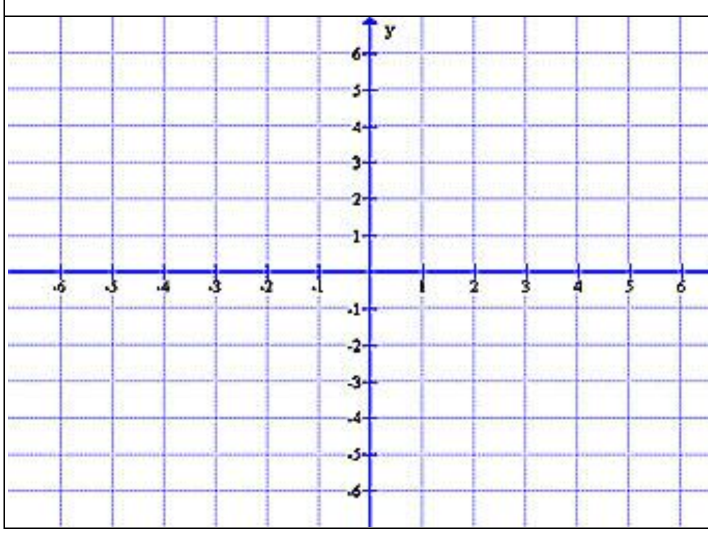
Blanks for practice with graphs

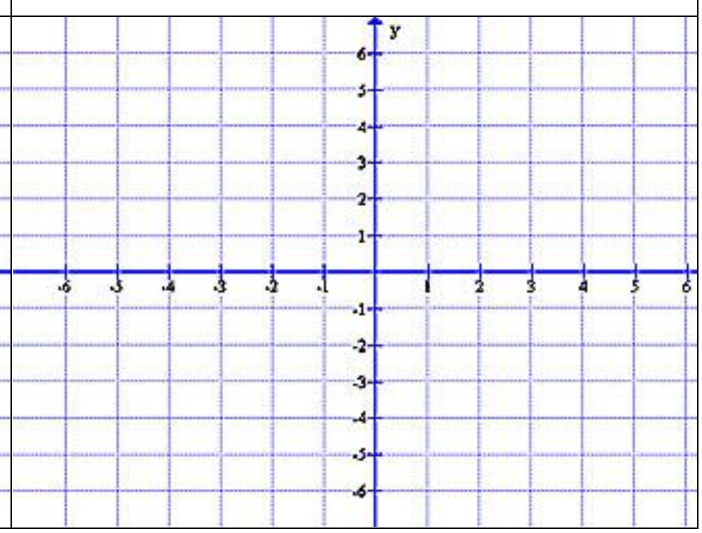


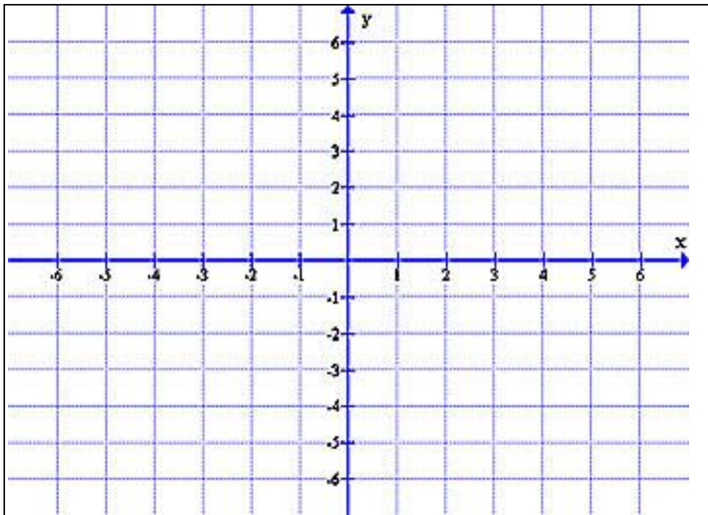
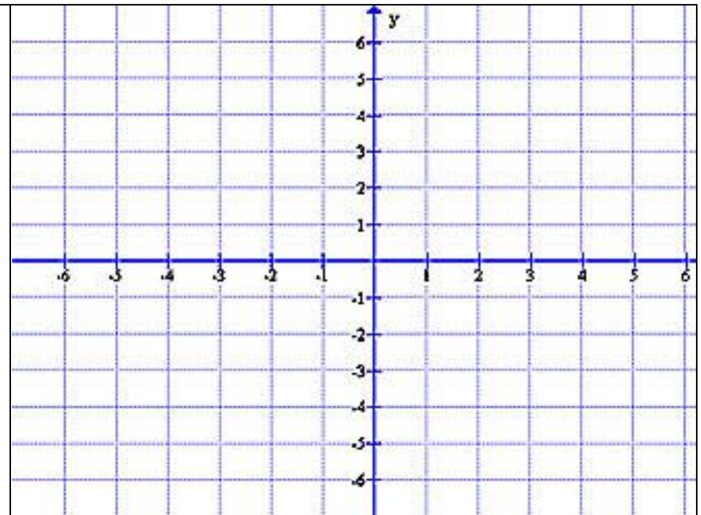
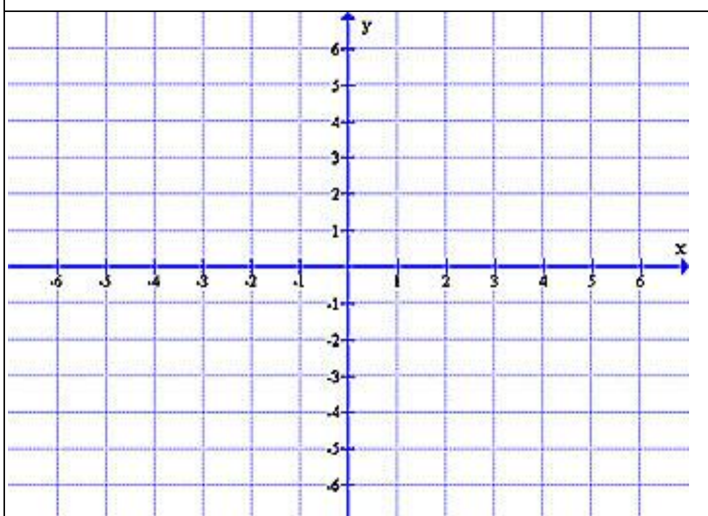
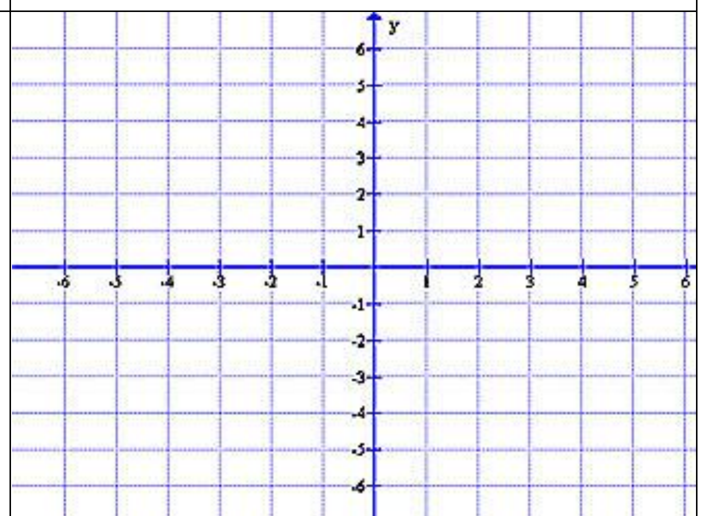
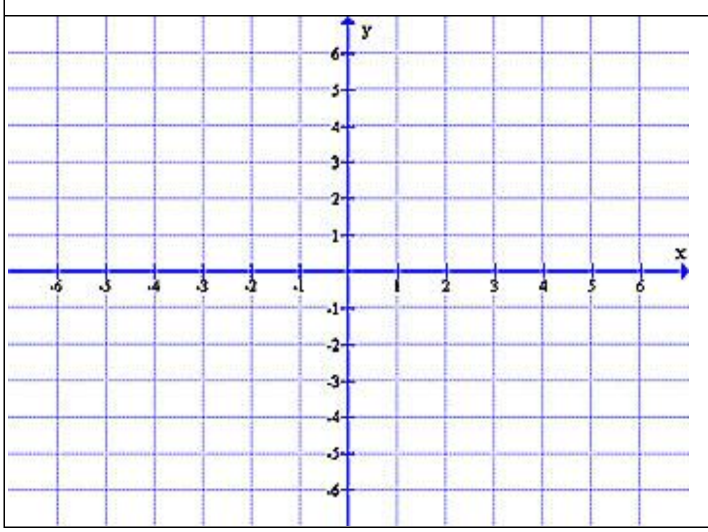
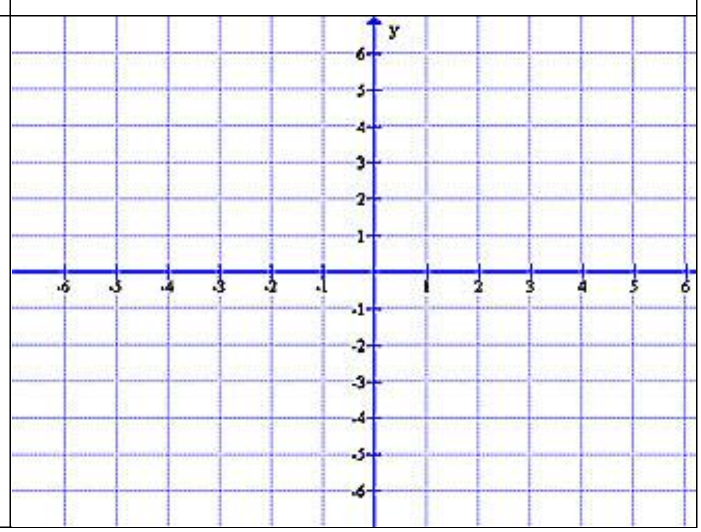


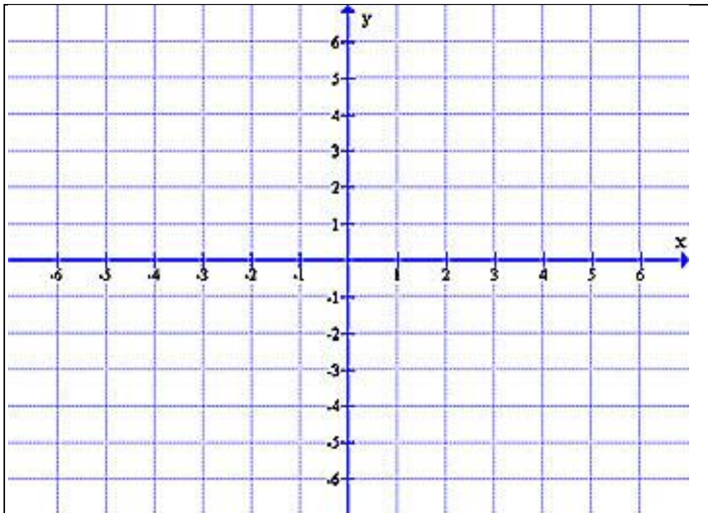






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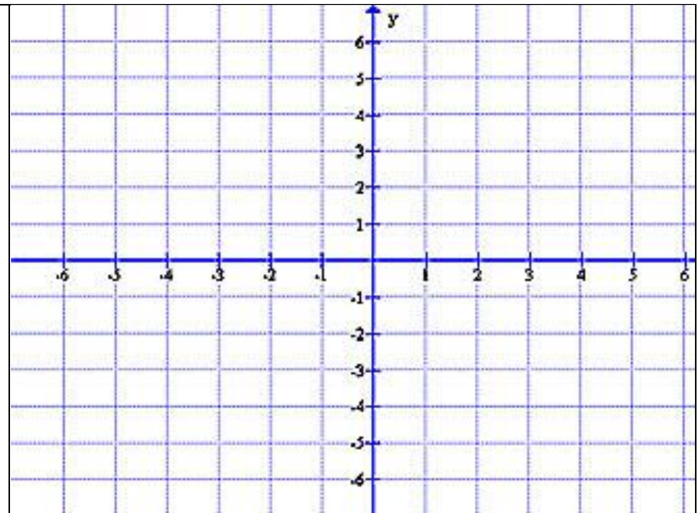
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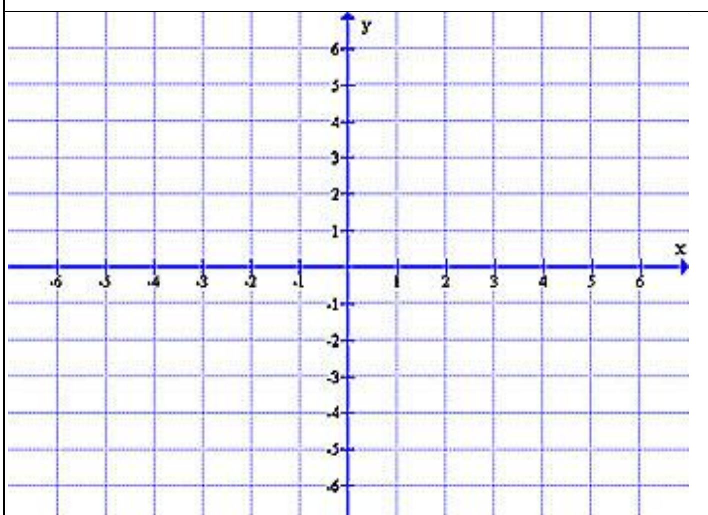
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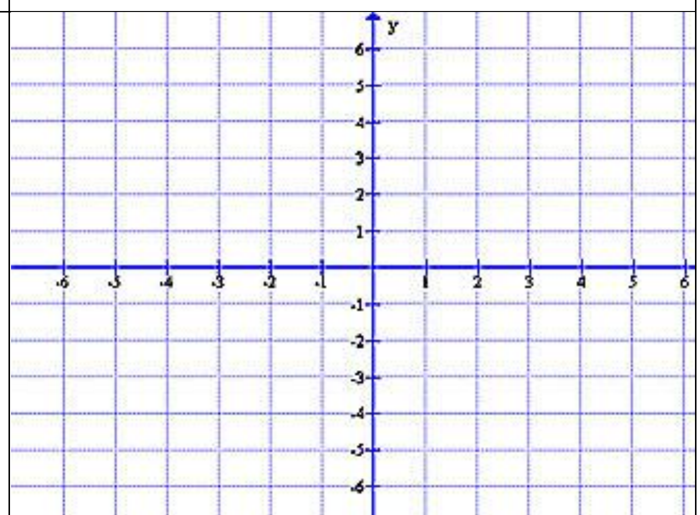
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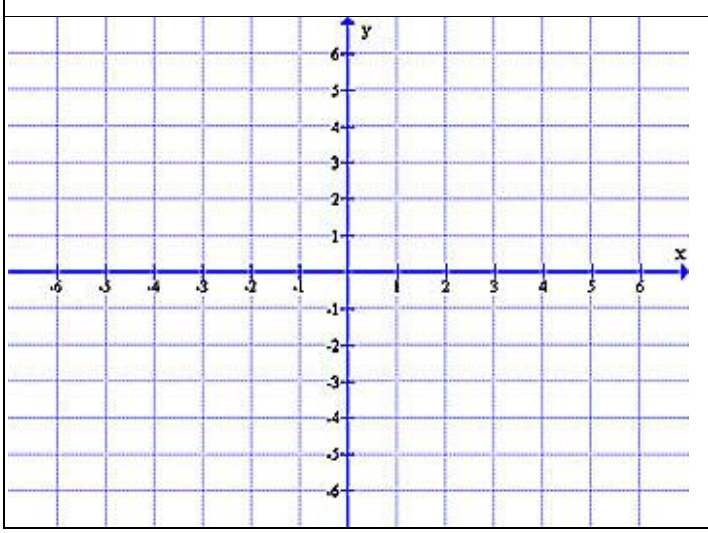
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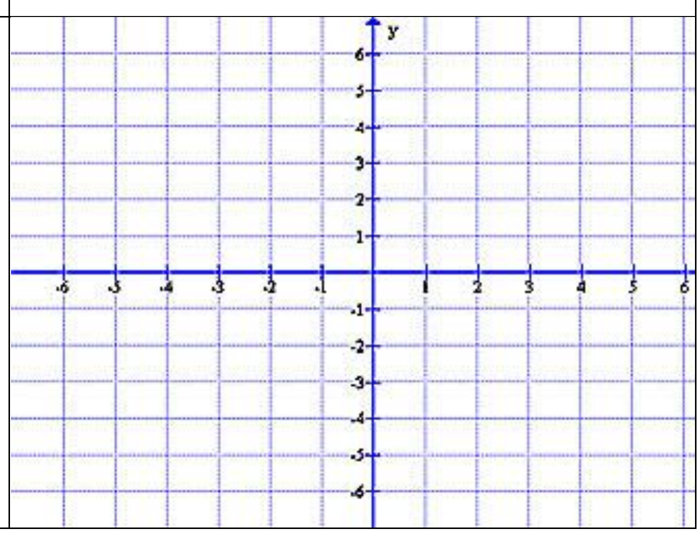
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