Math 1112

Unit 7: Probabilities in Simple Games of Chance

7.1 Calculate probabilities by counting outcomes in games of chance

1. List all possible outcomes in games of chance
2. Terminology
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4. One event continued
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7. Tree diagrams for 2 events (outcomes with equal likelihoods).
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11. odds.

List all possible outcomes in games of chance: use your ability to use idea of sets to think slowly about the following.

Exercise 1: List all elements of the following sets as defined as a list of possible distinct outcomes (i.e. within curly brackets {}) and find the cardinality of the set.
If you believe that the set is not well-defined simply state ‘not well-defined’ and describe what makes it ‘not well-defined’. Label each scenario as a single event or two event scenario.

a. Two coins are tossed at the same time (a nickel and a penny). Outcomes of interest: sides facing up.

b. Three coins are tossed at the same time (a quarter, a nickel and a penny). Outcome of interest: sides facing up.

c. You reach into a bag containing 7 marbles and grab one marble. 3 marbles are green, 3 are red and the last one is blue to draw one marble.

d. There are two marbles in a box, but you don’t know their colours. The only thing you are told is that ***if*** one is a red marble, ***then*** the other is a blue marble. Outcome of interest: colour of marbles in box. (Hint: you don’t need to list each possible colour.)

e. 2 six-sided dice are rolled and of interest are the numbers facing up.

f. 2 four-sided dice (triangular pyramids) are rolled and of interest are the numbers facing up (see example to the right).

Concepts and terminology

Calculating probabilities can be complex when the scenario presented is complex or ill-defined (like the probability that you will fall down the stairs before you turn 80 years old). Don't worry, that is not a scenario that we will try to model. We will start with straightforward scenarios where possible outcomes can be counted. The most straightforward scenarios are games of chance in which each outcome is equally likely (like the flip of a coin or the rolling of one or two dice). Surprisingly many scenarios which don't seem to have equally likely outcomes can be broken down and modeled using this approach.

<u>Terminology:</u>

phenomenon: an action, or series of actions (an event or series of events) where there may be more than one outcome. (formally referred to as 'experiment')

outcome: result of an action or series of actions within a well-defined phenomenon

sample space (*S.S.*): the set of all possible outcomes of a phenomenon

event (E): subset of the sample space containing one or more outcomes; can be referred to as the desired/favoured outcome(s)

describe S.S.: describe the contents of the set of possible outcomes of an experiment

list *S.S.* : list all possible outcomes in the sample space as a set

list an event E: list all elements that are considered successes, favoured or desired

independent events: two or more events (like coin toss and roll of die) that are not related – i.e. knowing the outcome of one event has no bearing on the other.

Mutually exclusive events: mutually exclusive events are dependent in that if events A and B are mutually exclusive, then if A happens this means that B cannot happen, thus B is dependent on A.

each outcome is equally likely to occur

Probability of E = P(E)

$$= \frac{\textit{number of outcomes in E}}{\textit{number of outcomes in S.S.}}$$

$$= \frac{n(E)}{n(S.S.)}$$ = cardinality of set E divided by cardinality of S.S.

Steps to calculating probability of an event E

i. List the sample space, and find n(S.S)

ii. List the event - desired outcome(s) - and find n(E)

iii. Calculate $P(E) = \frac{n(E)}{n(S.S.)}$

One event games of chance:

Example 1: **P(rolling an even number when rolling a fair 6 sided die)**

i. *S.S.* = {1,2,3,4,5,6} n(SS) = 6

ii. *E* = {2,4,6} n(E) = 3

iii. P(even number) = 3/6

Example 2: with eyes closed, draw one marble from a bag containing 7 red, 4 green and 3 blue marbles. Find P(green marble)

i. *Description of S.S.*: you will either draw a red, a green or a blue marble, but we need to treat each marble as distinct (there are 14 marbles not 3 – thus there are 14 possible outcomes, not 3. The possible outcomes are one of 7red, 4 green and 3 blue marbles

List of S.S. = {r,r,r,r,r,r,r,g,g,g,g,b,b,b} n(SS) = 14

ii. E = choosing a green marble from a bag contains 7 red, 4 green and 3 blue marbles.

E = {g, g, g, g} n(E) = 4

iii. P(choosing a green marble from a bag with 7R, 4G, 3B) = 4/14

Exercise 1: for each of the following follow the 3 steps to calculate the given probability. The scenario is a roll of one single sided dice.

a)P(>4) b) P(odd #) c) P(≥ 4)

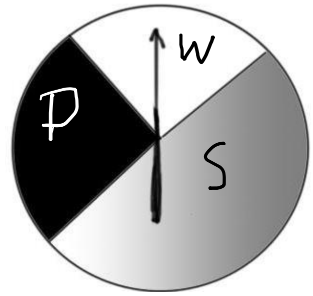
d) P(≤ 7) e) P(1 or 6) f) P(< 1)

g) P(odd # >2) h) P(even # ≤ 7) i) P(0)

One event games continued:

Exercise 2:

A circle is typically divided into sections called degrees; there are 360˚(degrees) in a circle. In the spinner to the right the white slice (W) takes up 60˚ out of a total possible 360˚, the dark (D) takes up 120˚ and the shady slice (S) takes up the rest.



If the spinner is fair, find the probability that the spinner will land on the following section by finding

a) P(W)

b) $P(W^C)$ i.e. P(anything but W)

c) P(D)

d) P(D or S)

e) D and S

f) P(W or D or S)

Exercise 3:

A jar contains 7 blue marbles, 14 yellow marbles, and 12 purple marbles. If you choose a marble randomly find the probability that:

a) the chosen marble is blue.

b) the chosen marble is purple.

c) the chosen marble is green.

d) the chosen marble is blue or green.

e) the chosen marble is red or yellow

f) the chosen marble is blue and yellow

g) the second ball chosen is blue, if the first ball chosen is blue, and you lose it

Two independent events games of chance

When working with 2 independent events the key is to understand the scenario (or the experiment) that is being described. This means carefully reading the instructions given and writing out all the possible outcomes (the Sample Space) first… slowly and methodically.

If you can't write out the sample space then you likely do not understand the scenario.

Exercise 1: List the sample space SS, and n(SS) for the following experiments.

a) Flip 2 fair coins

b) Flip a two sided, fair coin and roll one die.

Exercise 2: For the following experiments list the events E and n(E)

a) Flip 2 fair coins:
E = one head and one tail

b) Flip one coin and roll one fair die:
E = one head and an even number

Exercise 3: Find the following probabilities

a) P(one H & one T) flip 2 coins.

b) P(one head and an even number) flip a coin and roll one die.

Exercise 4: for the gold medal.

a) list the SS and n(SS) for flipping three coins

b) list SS and n(SS) for number of tails in a row when flipping a coin until heads appears

c) list SS and (n(SS) for rolling 2 dice

Two independent events: dice – exercises

Below is a chart that represents the outcomes that make up the sample space for one roll of two 6-sided dice.

Possible sums when rolling two dice

Die 1	**Die 2**					
	1	2	3	4	5	6
1	1+1=2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	3+4=7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example 1: Find the probability that you roll 11 when rolling 2 dice.

Step 1. List the sample space for the above 'experiment'. What is n(SS)?

S.S. = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

n(SS) = 36

Step 2. Event E = a sum of 11; E = {(5,6) (6,5)} n(E) = 2

Step 3. P(E) = 2/36 = 1/18

Example 2: Find the probability that you roll a sum of 7 when rolling 2 dice.

n(SS) = 36 (from above)

Event E – getting a sum of 7; E= {(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)} n(E) = 6

P(E) = $\frac{n(E)}{n(SS)} = \frac{6}{36} = \frac{1}{6}$

Example 3: Find the probability of rolling a sum of 7 or an 11 when rolling 2 dice

Event E – getting a sum of 7 or 11; E= {(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)(5,6)(6,5)} n(E) = 8

P(E) = $\frac{n(E)}{n(SS)} = \frac{6+2}{36} = \frac{8}{36} = \frac{2}{9}$

<u>Exercise 1:</u> in rolling two dice find

a) P(sum=6)

b) P(sum>6)

c) P(sum is odd)

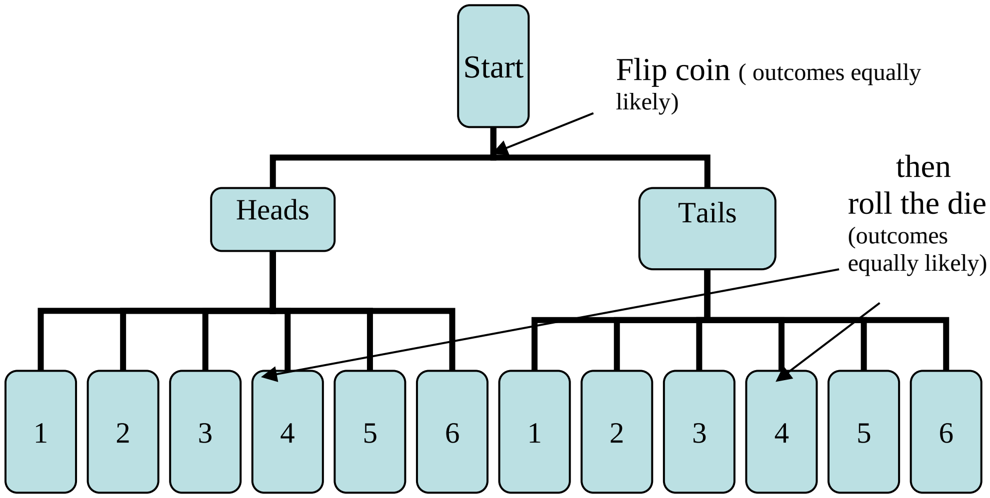
d) P(sum is 14)

e) P(sum is 12 or 2)

f) P(sum is ≤ 7)

Probability trees (equal probabilities of outcomes)*:* are another way of modeling 2 events where chance plays a role.

The probability tree below models the experiment: tossing of one coin followed by the rolling of a die, which we investigated a few pages back. (Note that in this example outcomes at each stage are equally likely.)



S.S.={ H1, H2, H3 H4, H5, H6, T1, T2, T3, T4, T5, T6}

n(SS) = 12

Example 1: Find the probability of the event E = 'Heads and a 1 or H1 for short'.

P(E) = $\frac{n(E)}{n(SS)} = \frac{1}{12}$ - H1 exists only once and the sample size has 12 events

Example 2: Find the probability of the event 'heads or tails and a 1'.

P(E) = $\frac{n(E)}{n(SS)} = \frac{2}{12} = \frac{1}{6}$

Exercise 1: Find the following probabilities for the above scenario:

a) P(H)

b)P(Head and even number)

c) P(Head or even number)

d) P(H3 & T3)

e) P(Head or Tail and a 3)

f) P(Tail and >3)

Equal probability tree exercise

<u>Exercise 2:</u> consider a rather strange game involving the flipping of a coin followed by spinning a spinner with 3 colour s (green, yellow and blue) with each having equal likelihood of appearing.

a) sketch the probability tree representing the strange game below

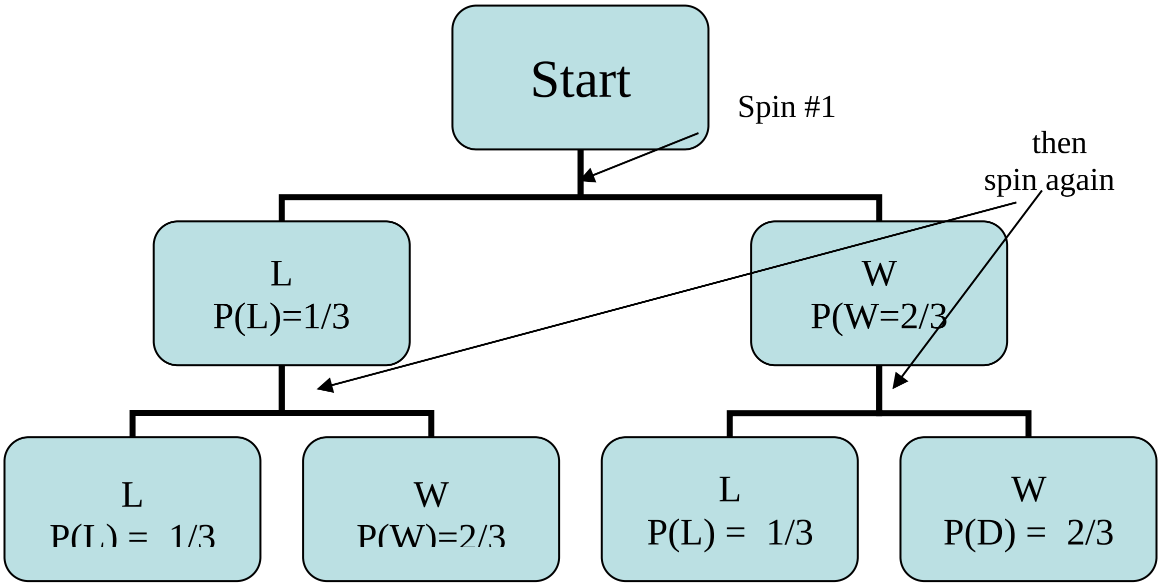
b) use the tree diagram to calculate P(Heads and yellow)

c) use the tree diagram to calculate P (Tails and green)

d) use the tree diagram to calculate P(Heads and blue or yellow)

Probability trees (unequal probabilities of outcomes): modeling events where chance plays a role and the outcomes don't have equal probability.

Example 1: The probability tree below models two spins of a coloured spinner in which the probability of L = 1/3 and the probability of W = 2/3. Though there are two possible outcomes at each spin, because they do not have equal probability, we need to account for the likelihood by creating a 'fake' sample space with equally likely outcomes. The tree diagram sorts out the probabilities at each stage and the expanded 'fake' sample space accounts for the probabilities accurately.



S.S.= {LL, LW, LW, WL, WL WW, WW, WW, WW}

n(SS) = 9

Use the tree diagram above to find the following probabilities for the result of an experiment in spinning the above spinner twice.

a) P(getting L twice)
n(SS) = 9
n(LL) = 1
P(LL) = 1/9

b) P(getting L once and D once)
n(SS) = 9
n(LW or WL) = 4
P(LW or WL) = 4/9

Exercise 1: use the information above to compute the following probabilities (L = lose; W = win)

a) P(win both)

b) P (lose both)

c) P(win game1 or win game 2) - include win both in 'or'.

Probability Tree exercise

Exercise 1 ***(challenge question):*** set up a tree diagram for the following and find the probabilities
Given P(win game1) = 4/5; P(win game 2) = 2/3

a) Sketch the probability tree below

b) P(win both) =

c) P (lose both) =

d) P(win game1 or win game 2)

Probability: **Odds**

Odds are often confused with probabilities. They are related, but not the same thing. The odds of an event (E) happening is simply the ratio of the probability of E {P(E)} over the probability the E will not happen {P(E^C)}

Odds of E = $\frac{P(E)}{P(E^C)}$ therefore Odds against E = $\frac{P(E^C)}{P(E)}$

Odds of E tell you to what extent E is more likely than E^C. In the health care system it is often the preferred way of expressing the one's likelihood of getting ill.

Example 1: P(E) = 4/19, what are the odds of E

P(E) = 4/19, P(E^C) = 15/19 then Odds of E = $\frac{\frac{4}{19}}{\frac{15}{19}} = \frac{4}{15}$ = 4:15 i.e. for every 4 people who get E there are 15 who do not.

note that the sum of the odds (4 + 15 = 19 = the sample space)

Example 2: The odds of getting sick if exposed to West Nile virus are 2:8. Find P(getting sick if exposed to West Nile virus) = P(S given WN).

Visualize this by rewriting the odds of getting sick given WN = 2:8 = $\frac{2}{10} : \frac{8}{10}$

therefore P(S given WN) = $\frac{2}{10}$, P(not getting sick given WN) = $\frac{8}{10}$

Exercise 1: Complete the chart below.

Description of event E	P(E)	P(E^C).	Odds of E
a Heads in flip of one coin			
b >3 in rolling one die			
c Getting diabetes if one ever smoked daily (exercise 1 pg. 18)			
d 7 or 11 in rolling of 2 dice			
e At least one head in flipping two coins			

Exercise 2: Find probabilities of the following events given the odds.

a) Odds of getting disease Q = 4:5 Find P(Q)
b) Odds of getting disease R = 4:1 find P(R)
c) Odds of getting disease S = 1:3 Find P(S^C)
d) Odds of getting disease T = 5:2 Find P(T^C)
e) Odds of getting disease U = 2:2 Find P(U)